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First-order Scattering Effects on Millimeter-wave Spaceborne Radar Reflectivity Measurements: a Model Approach

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1 Introduction

Microwave meteorological (or weather) radars represent a well established technique to retrieve the rainfall structure and microphysics (Sauvageot, 1992; Doviak and Zrnic, 1993; Bringi and Chandrasekar, 2001). Due to constraints on the component sizes (e.g., waveguides, power transmitter and antenna reflector) space-based sensors operating at high, attenuating frequencies (i.e., Ku to W bands) have been taken into consideration for cloud and rain retrieval (Meneghini et al., 1989; Austin and Stephens, 2001)].

From a theoretical point of view, the meteorological radar equation in attenuating media is generally formulated, in its classical form, by assuming single-scattering and by taking into account the two-way path attenuation through a uniform plane-wave model (Sauvageot, 1992); Doviak and Zrnic, 1993; Bringi and Chandrasekar, 2001). The radiative transfer theory, on the other hand, deals with absorption and scattering of electromagnetic radiation in tenuous media. The relation between the classical radar equation and the radiative transfer theory is not only expected, but also intuitive. However, even though this relation is suggested in some fundamental works (Ishimaru, 1978; Tsang et al., 1985), it has not necessarily stated by carrying out an exhaustive analytical exercise within the radar meteorology framework.

In this work, the relation between the radiative transfer and the radar equation approaches is rigorously demonstrated, showing that the two formulations lead to the same analytical solution under the first-order multiple scattering approximation. In order to derive this result in Sect. 2, a generalized definition of the radar reflectivity in terms of the received specific intensity is introduced within the radiative transfer framework and the solution derived for a stratified attenuating media. On the other hand in Sect. 3, the integral form of the radar equation is solved under simplified assumptions to show how its analytical solution is identical to the first-order scattering one. Interestingly, contribution of the single scattering effects in the radar equation budget is clearly separated from those due to the two-way path attenuation, by introducing a *radar range-bin factor* ρ_{rb},. In Sect. 4 a numerical analysis of this radar range-bin factor leads to identify the accuracy of approximated formulas for expressing the radar equation in attenuating media. This quantitative evaluation is carried out considering frequencies from Ka to W band and employing statistical radar relationships for rainfall observations.

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2 Radiative Transfer Approach

Scattering and propagation characteristics of microwaves through a random medium can be profitably studied by using the *radiative transfer theory* (RTT) (Ishimaru, 1978; Tsang et al., 1985). The fundamental quantity in RTT is the *specific intensity I* (also called *radiance*). The specific intensity of a radiation of frequency ν is defined as the average power flux density within a unit frequency band centered at ν and within a unit solid angle and is consequently measured in [$Wm^{-2}sr^{-1}Hz^{-1}$].

In this section we first to introduce the definition of the *apparent* radar reflectivity in terms of the radar received and transmitted specific intensity. Once defined the apparent radar reflectivity, the received power can be calculated in a straightforward manner. An explicit form of the received power in a stratified medium, from a radiative transfer point of view, can be derived by assuming a first-order multiple scattering solution which can be obtained by adopting an iterative method.

2.1 Apparent radar reflectivity

We consider a radar antenna placed in the origin of a spherical coordinates system (r,θ,ϕ), which transmits a power P_T along the direction $\Omega_0=(\theta_0,\phi_0)$. Of course, when $\theta_0=0$ and $\phi_0=0$, $\Omega_0=(0,0)$ indicates a nadir observation.
The transmitted power flux density $F_T(r)$ at range r in the forward direction $-\Omega_r$ is given by (Ishimaru, 1978):

$$F_T(r) \equiv \frac{P_T G(\Omega_r)}{4\pi r^2}, \quad (1)$$

where $G(\Omega_r)$ is the antenna gain function with its maximum along the pointing angle Ω_0.

Zebker et al. (1990) define the *apparent radar reflectivity* η_a as the ensemble average of backscattering cross sections of all the particles within a unit volume. Marzano et al. (2000) give a formulation of the *apparent radar reflectivity* within the radiative transfer theory as follows:

$$\eta_a(r) \equiv \frac{4\pi}{\Delta r}\frac{< I_R(r,\Omega_0) >}{F_T(r)}, \quad (2)$$

being $I_R(r,\Omega_0)$ the backscattered specific intensity at range r in the Ω_0 direction [Ishimaru (1978)], $\Delta r = c\Delta t/2$ the radar range resolution, with Δt the pulse width, and c the light velocity (in vacuum).

In (2) the ensemble average indicates the averaging processing, within the resolution volume ΔV_r, of all particle states in time and space. The *apparent reflectivity* η_a can be related to the *apparent reflectivity factor* Z_a by means of the usual relationship:

$$Z_a(r) \equiv \frac{\lambda^4}{\pi^5 |K|^2}\eta_a(r). \quad (3)$$

Given the mean value $<I_R(r,\Omega_r)>$ of the *apparent* received specific intensity, the *apparent* backscattered received power $<P_{Ra}(r)>$ along the pointing direction Ω_0 is given by (Ishimaru, 1978):

$$< P_{Ra}(r) > \equiv \frac{\lambda^2}{4\pi}\int_{4\pi} G(\Omega_r) < I_R(r,\Omega_r) > d\Omega_r. \quad (4)$$

Thus, substituting (1), (2) and (3) in (4), we obtain a *generalized* radar equation:

$$< P_{Ra}(r) > \cong C\frac{Z_a(r)}{r^2}, \quad (5)$$

with the *radar instrumental constant* C that is equal to

$$C \equiv \frac{P_T \pi^2 |K|^2 G_0^2 \Omega_{2A} \Delta r}{64\lambda^2}. \quad (6)$$

In the above equation, Ω_{2A} the antenna two-way radiation solid angle and λ is the wavelength.

Equation (5) is formally identical to the *classical* radar equation when the *apparent* reflectivity factor Z_a is equal to the measured reflectivity factor. However, the *generalized* radar equation (5) has a different physical meaning and a more general validity with respect to the *classical* radar equation since it can take into account multiple scattering effects to any order, being the apparent radar reflectivity related to specific intensity solution of the radiative transfer equation (Marzano et al., 2000, 2003).

2.2 First-order multiple scattering

Under the assumption of unpolarized (or linearly polarized) radiation, the specific intensity of the scattered radiation is solution of a scalar differential-integral equation, known as radiative transfer equation, or RTE (Ishimaru, 1978).

An explicit expression of Z_a, given in (3) can be obtained if we limit our analysis to the first-order multiple scattering (FOS) following an iterative solution method for RTE. In fact, when first-order scattering is the dominant contribution (i.e., tenuous medium with predominant absorbing effects, that is relatively small volumetric albedo) the *apparent* radar reflectivity factor for a slab of depth $\Delta r=\Delta z/\mu_0$ is given by (Marzano and Ferrauto, 2003):

$$Z_{aFOS}(r) = Z_e(r)\frac{\left(1-e^{-2\Delta\tau_r}\right)}{2\Delta\tau_r}, \quad (7)$$

with Z_e that is the well-known equivalent reflectivity factor. Looking at the second right-hand side of (7), it appears convenient to define a *range-bin extinction factor* $f_b(r)$:

$$f_b(\Delta r) \equiv \frac{1-e^{-2\Delta\tau_r}}{2\Delta\tau_r}, \quad (8)$$

which takes into account the global extinction effects within the range bin of resolution Δr due to first-order multiple scattering.

So, as a particular case of (5), the FOS radar equation is given by:

$$< P_R(r) >_{FOS} = C\frac{Z_{aFOS}(r)}{r^2}L^2(r), \quad (9)$$

where $L^2(r)$ is the two-way path attenuation, while

$$Z_{a\,FOS}(r) \equiv Z_e(r) f_b(\Delta r). \quad (10)$$

It is important to notice that previous results have been obtained for a single homogeneous slab. The latter equation will be also deduced by following the radar equation approach, as shown in the next section.

3 Radar Equation Approach

In the next two paragraphs we will show how to derive the *generalized* radar equation, starting from the *integral form* of the radar equation (hereafter also referred to as *integral* radar equation).

3.1 Integral radar equation

In presence of an inhomogeneous attenuating medium, the *integral* form of the radar equation can be stated as follows (Sauvageot, 1992):

$$< P_R(r_0) >= \frac{P_T \lambda^2}{(4\pi)^3} \int_{\Delta V_r} \eta(r,\Omega_r) \frac{G^2(\Omega_r)}{r'^2} e^{-2\tau_r(r)} \, dV_r \tag{11}$$

where λ is the radar wavelength (in vacuum), η is the volumetric radar reflectivity, Ω_r is the observation solid angle, τ_r is the optical thickness along the range r, dV_r is the elementary volume within the radar bin and r_0 is the range up to the centre of the radar resolution volume. The one-way path attenuation factor in (11) can be expressed as:

$$L(r) = e^{-\tau_r(r)} = e^{-\int_0^r k(r')dr'} \;, \tag{12}$$

being k the volumetric specific attenuation (or extinction coefficient). The integration in (11) can be extended to the finite range bin Δr (i.e., range resolution) as follows:

$$\begin{aligned} < P_R(r_0) > &\cong \frac{P_T \lambda^2}{(4\pi)^3} \frac{\pi^5 |K|^2}{\lambda^4} G_0^2 Z_e(r_0) \Omega_{2A} \times \\ & e^{-2\tau_r(r_0-\Delta r/2)} \int_{r_0-\Delta r/2}^{r_0+\Delta r/2} \frac{e^{-2\tau_r(r'-r_0+\Delta r/2)}}{r'^2} dr' \end{aligned} \tag{13}$$

where the effects of the range-weighting function have been disregarded and the radar antenna has been considered sufficiently directive such that the equivalent reflectivity factor Z_e becomes independent from the observation angle Ω_r and range r' within the radar volume. The last integral in (13) can be calculated only by assuming a known relationship of τ_r with respect to r'. Let us assume for simplicity a uniform specific attenuation k within each range bin. This implies that $\tau_r(r')= kr'$. By performing an integration by parts, the integral in (13) reduces to:

$$\int_{r_0-\Delta r/2}^{r_0+\Delta r/2} \frac{e^{-2kr'}}{r'^2} e^{2k(r_0-\Delta r/2)} dr' \cong \left(\frac{1-e^{-2k\Delta r}}{2k\Delta r} \right) \frac{\Delta r}{r_0^2} \tag{14}$$

The same result can be obtained by expressing the integral in (13) by means of the exponential integrals and using their large-argument expansion truncated to $1/r_0^2$, as derived by Meneghini et al. (1983).

Thus, for large distances from the radar with respect to the range resolution and uniform specific attenuation along the range, the mean received power from an arbitrary range bin can be rearranged as follows:

$$\begin{aligned} < P_R(r_0) > &\cong \left(\frac{P_T \pi^2 |K|^2 G_0^2 \Omega_{2A} \Delta r}{64 \lambda^2} \right) \times \\ & \frac{Z_{aFOS}(r_0)}{r_0^2} L^2(r_0 - \Delta r/2) \end{aligned} \tag{15}$$

where $Z_{aFOS}(r_0)$, already defined in (10), is the *apparent radar reflectivity factor* when first-order multiple scattering phenomena predominate. By using the convention to define the path integrated attenuation L up to the centre of each bin volume (i.e., the range r_0), the mean received power can be cast into the following *modified* radar equation as:

$$< P_R(r_0,\Omega_0) > \cong C \frac{Z_e(r_0,\Omega_0) \rho_{rb}(\Delta r)}{r_0^2} L^2(r_0) \;, \tag{16}$$

where the *radar range-bin factor* ρ_{rb} is defined as:

$$\rho_{rb}(\Delta r) = f_b(\Delta r) e^{k\Delta r} \;. \tag{17}$$

Equations (16)-(17) coincide with the *generalized* radar equation (9), derived under first-order scattering assumption, thus demonstrating the relation between the classical radar equation and the radiative transfer theory.

Notice that difference between the factors f_b and ρ_{rb} stated by (17) is misleading, since it spring from the usual operative convention to define the path integrated attenuation L up to the centre of each bin. Consequently, the ρ_{rb} factor expresses the first-order scattering effects on the radar signal propagation within the radar range bin of resolution Δr net of the extinction effects due to path attenuation, that are cancelled by the multiplicative term $exp(k\Delta r)$. Notice that (16) returns the *classical* radar equation in attenuating media, as a particular case, when ρ_{rb}=1 (or ρ_{rb}=0 dB).

A numerical analysis of the radar range-bin factor ρ_{rb} will be then shown in the next section to identify the accuracy of approximated formulas of the radar equation in attenuating media, conventionally used in several applications.

4 Numerical Analysis

The analysis of the range-bin effects can give some insights into the accuracy of radar equation expressions used in some meteorological applications. As previously mentioned, airborne and spaceborne radars have been proposed, and possibly deployed, at Ka and W bands. In order to approach more realistic cases, the attenuating medium has been modelled in terms of reflectivity and specific attenuation by power-law relations between equivalent reflectivity factor Z_e, specific attenuation k and rainfall rate R. We have selected Z_e-R and k-R expressions for Ka and W frequency bands relative to tropical rainfall scenarios from Haddad et al. (1997) (their Tables IX-XII for D”=1.0, with s”=0.39).

As a matter of fact, we report ρ_{rb} curves with R variability limited at 20 mm/h and 10 mm/h for Ka and W band, respectively. The range bin resolution Δr, in accordance with common operational requirements, has been varied from 250 m to 1 km.

Top panels of Fig. 1 illustrate the impact of the radar range-bin factor ρ_{rb} on the computation of the *apparent* radar reflectivity Z_a from (16), for the chosen frequency bands. We remind that in case of negligible effects, it should be ρ_{rb}=1 (or ρ_{rb}=0 dB).

Furthermore, bottom panels of Fig. 1 show fractional error $\Delta R/R$ in the rainrate estimation when the range-bin factor ρ_{rb} in (16) is neglected. A perturbation approach on the power-law relations Z_e-R and k-R is adopted to calculate the fractional error, when considering ρ_{rb} as a multiplicative noise (or, if expressed in dB, as an additive noise).

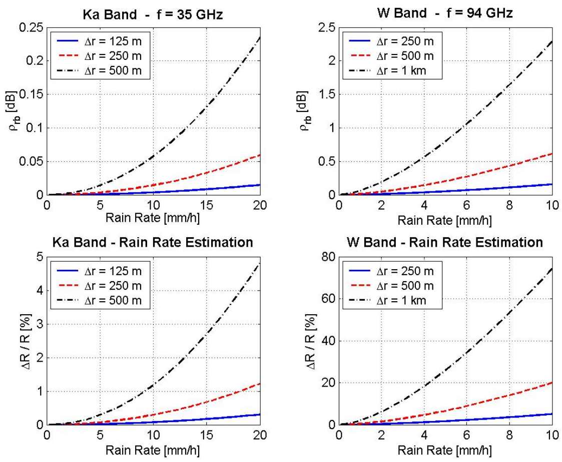

Fig. 1. *Top panels*: Radar range-bin factor ρ_{rb}. at Ka and W band. *Bottom panels*: Rainrate fractional error $\Delta R/R$, derived from the analysis of the approximated radar equation, at Ka and W band.

At Ka band, ρ_{rb} keeps under 0.25 dB also for R=20 mm/h and 1 km range resolution and its impact on the radar reflectivity estimation can be considered negligible, since the fractional error in the R estimation keeps lower than 5%.

Range-bin single scattering effects become significant for frequencies above Ka band. At W band ρ_{rb} even reach values of 0.5-1.5 dB at 8 mm/h and the fractional error $\Delta R/R$ can be higher than 20% for a radar range resolution above 500 m.

5 Summary and Conclusions

The relation between the radiative transfer and the radar equation has been rigorously demonstrated, showing that the two approaches lead to the same analytical solution under the approximation of the first-order multiple scattering solution. A generalized expression of the radar equation in attenuating media, which can take into account multiple scattering effects to any order, has been derived.

First-order scattering solution shows that a multiplicative factor, here called *radar range-bin factor*, should be added to the *classical* radar equation in attenuating media.

A quantitative evaluation of this factor has been carried out at Ka and W bands to identify the accuracy of approximated formulas for the radar equation in attenuating media. For frequencies above Ka band, the *radar range-bin factor* shall be properly taken into account in the rainrate estimation, in order to avoid fractional errors higher than 20%.

Finally, first-order multiple scattering solution is acceptable when considering a predominantly absorbing medium such rainfall at microwaves. In other cases, where the volumetric albedo is significantly high (i.e., graupel and hail), the *generalized* radar equation, derived from the RTT, should be applied to radar data analysis in order to tackle apparent received power due to a combination of reflectivity, path attenuation and high-order multiple scattering.

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