Extensive comparison of various candidates for operational attenuation correction with a C-band polarimetric radar

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1 Introduction

Quantitative use of weather radar measurements (i.e., precipitation estimation, hydrometeor classification) requires accurate evaluation of the main error sources such as calibration, ground clutter, anomalous propagation, beam blockage and rain path attenuation (e.g., Bringi and Chandrasekar, 2001). At frequencies higher than S band, the latter becomes significant and needs to be compensated. When attenuation occurs, the measured reflectivity at horizontal polarization \( Z_h^' \) [mm\(^6\) m\(^{-3}\)], differential reflectivity \( Z_{dr}^' \) (expressed in linear units) can be written as

\[
\left( Z_h^' - Z_{h,dp}^0 \right) = \int_0^\alpha h_s \, d\Phi_{h,dp} \, ds
\]

where \( \alpha_h \) and \( \alpha_{dp} \) are the specific attenuation and differential attenuation, respectively. Since the beginning of radar meteorology, several approaches for attenuation correction have been suggested in the literature (Hitschfeld and Bordan, 1954; Testud et al., 2000; Vulpiani et al., 2005; Gourley et al., 2006). The aim of the present work is to propose a new approach and compare it with other techniques currently available in the literature. Several cases, observed during the summer 2005 in the Paris area by the dual-polarized weather radar operating in Trappes, are analyzed and discussed.

2 Advanced polarimetric techniques for attenuation correction

As scattering simulations have demonstrated (Bringi et al., 1990; Jameson, 1992), specific attenuation and differential attenuation (\( \alpha_h \) [dB km\(^{-1}\)]) are almost linearly related to specific differential phase (\( K_{dp} \) [° km\(^{-1}\)]) when \( D_0 \leq 2.5 \text{ mm} \) (where \( D_0 \) is the median volume diameter)

\[
\alpha_{h,dp} = \gamma_{h,dp} K_{dp}
\]

\( \gamma_{h,dp} \) being dependent on drop size distribution, drop shape and temperature. Starting from this paradigm, a simple attenuation correction technique, based on the relation between differential phase \( \Phi_{dp} \) and cumulative attenuation (differential attenuation) \( A_{h,dp} \), has been proposed by Bringi et al. (1990) and evaluated by other authors (i.e., Carey et al., 2000). Assuming \( \gamma_{h,dp} \) constant in range, \( A_{h,dp} \) [dB] can be expressed as

\[
A_{h,dp}(r) = \gamma_{h,dp} K_{dp}(r) - \Phi_{dp}(r) + A_{h,dp}(r) = \gamma_{h,dp} K_{dp}(r) - \Phi_{dp}(r) + \frac{\gamma_{h,dp}}{2} \Delta \Phi_{dp}(r, r_0)
\]

Consequently, the corrected reflectivity (differential reflectivity) becomes

\[
10 \log_{10} \left( Z_{h,dp}(r) \right) = 10 \log_{10} \left( Z_{h,dp}(r) \right) + 2 A_{h,dp}(r) = 10 \log_{10} \left( Z_{h,dp}(r) \right) + \gamma_{h,dp} \Delta \Phi_{dp}(r, r_0)
\]

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2.1 Rain Profiling algorithms

For single-polarized radars, the iterative approaches to rain path attenuation correction, beginning from the first range resolution volume and proceeding forward to successive resolution volumes, are known to be unstable if path-integrated attenuation is relatively large (Hildebrand, 1978). A notable improvement to the design of attenuation correction procedures has come from using the total path-integrated attenuation (PIA) as a constraint. Using the notation introduced in (3), the PIA can be written as

\[ \text{PIA} = A_d(r_n) \]

where \( A_d \) and \( r_n \) being the range distance at which the rain cell ends. The rain profiling algorithms, first developed for spaceborne weather radar, are a family of analytical solutions of the differential equation obtained from (1) when expressed as:

\[ \frac{\partial \rho}{\partial r} + \gamma = 0 \]

where \( \gamma \) is linearly related (\( \rho = aZ_h^b \)), the estimated PIA being assumed as boundary condition (Marzoug and Amayene, 1994; Iguchi and Meneghini, 1994, Vulpiani et al., 2006). A part from the Final Value method (FV), rain profiling algorithms also include the so-called Attenuation-Adjustment (AA) and Constant-Adjustment (CA). The AA method introduces a correction factor to adjust the coefficient \( a \) while CA, which assumes that \( a \) is constant in range, adjusts the solution for the radar constant, resulting independent from the calibration error. The so-called ZPHI algorithm proposed in Testud et al. (2000), represents the polarimetric version of the rain profiling algorithm named CA making use of the differential phase shift to estimate the PIA (by means of (3)). A self-consistent scheme to improve the ZPHI method, taking into account the temperature and shape dependency of \( \gamma_{h,dp} \), has been proposed by Bringi et al. (2001). In the present work, we focused on the Final Value method (FV) and CA (ZPHI) techniques using the \( \Phi_{dp} \) constraint to estimate the PIA. Furthermore, the approach suggested in Bringi et al. (2001) is adopted to retrieve the optimal values of \( \gamma_h \). As mentioned before, \( \alpha_0 \) and \( \alpha_{dp} \) are linearly related (\( \alpha_{dp} = \alpha_0 \) at C band. Consequently, once the range profile of \( \alpha_0 \) is obtained from \( \Phi_{dp} \) and ZPHI, the corresponding one of \( \alpha_{dp} \) can be easily derived estimating the path integrated differential attenuation (\( \text{PIDA} = A_d(r_n) \)) as PIDA = \( \alpha_0 \) PIA.

2.2 Optimized \( \Phi_{dp} \) method

An optimized version of the basic \( \Phi_{dp} \)-based method (hereinafter OPDP) is proposed in this work. As mentioned before, \( \gamma_{h,dp} \) are sensitive to shape and size distribution of raindrops. A preliminary classification of different rain conditions (i.e., light rain, medium rain, heavy rain, and large drops) would enable the use of suitable values of \( \gamma_{h,dp} \) obtained from scattering simulations or disdrometer measurements. For such purpose, the estimated \( \gamma_{h,dp} \) is used here, being unaffected by attenuation. The Maximum a Posteriori Probability criterion (MAP) is applied to classify the different rain conditions. Using Bayes theorem, the conditional probability density function (PDF) of a considered rain class \( c \), given a measurement \( K_{dp} \), can be expressed as:

\[
p(c \mid K_{dp}) = \frac{p(K_{dp} \mid c)p(c)}{p(K_{dp})} = \frac{p(\Delta K_{dp}^{(c)})p(c)}{p(K_{dp})}
\]

where \( \Delta K_{dp}^{(c)} = K_{dp} - K_{dp}^{(c)} \), \( K_{dp}^{(c)} \) [\( ^{o} \text{km}^{-1} \)] is the perturbation of specific differential phase measurements from the mean value \( K_{dp}^{(c)} \) of class \( c \) and \( p(c) \) represents the a priori discrete probability of class \( c \). The MAP estimation of rain class \( c \) corresponds to the following maximization with respect to \( c \):

\[
\hat{c} = \text{Mode}_c \left[ \ln p(c \mid K_{dp}) \right] (6)
\]

where \( \text{Mode}_c \) is the modal value of PDF with respect to \( c \). If \( p(\Delta K_{dp}^{(c)}) \) is assumed to be a Gaussian PDF, then (6) reduces to:

\[
\hat{c} = \text{Mode}_c \left[ -\frac{\left( K_{dp} - \langle K_{dp}^{(c)} \rangle \right)^2}{\sigma_{k_{dp}}^2} - \ln[\phi^{(c)}] + 2 \ln p(c) \right] (7)
\]

where \( \sigma_{k_{dp}} \) [\( ^{o} \text{km}^{-1} \)] is the specific differential phase standard deviation of class \( c \) and the rain classes have been assumed uncorrelated. Computing (7) requires knowledge of the specific differential phase mean \( \langle K_{dp}^{(c)} \rangle \) and standard deviation \( \sigma_{k_{dp}} \) of each rain class. In this work, the statistical characterization of each rain class has been derived from scattering simulations according to the rain parameterization described in Straka et al. (2000). The prior probability \( p(c) \), which has been assumed uniform, could be used to subjectively weight each class as a function of other available information (i.e., numerical models, disdrometer measurements). Once the classification is performed at each range gate \( r_n \), the following \( K_{dp} \)-weighted average of \( \gamma_{h,dp} \) is used for the whole range profile:

\[
\gamma_{h,dp}(r) = \frac{\sum_i K_{dp}(r_i) \gamma_{h,dp}^{(c)}(r_i)}{\sum_i K_{dp}(r_i)}
\]

where \( \gamma_{h,dp}^{(c)}(r_i) \) is the value of \( \gamma_{h,dp} \) corresponding to the rain class \( c \) detected at the range distance \( r_i \). The reason of this choice is the need to properly weight the values of \( \gamma_{h,dp} \) according to their contribution to the rain path attenuation.

3 Numerical Results

Quantitative validation of attenuation correction procedures is a cumbersome task. The comparison between radar rainfall estimate and rain gauges measurements is prone to several uncertainties such as the raindrop size distribution variability which affects the radar rainfall retrieval by means of a fixed Z-R relationship. In the present work, the physically based approach proposed by Smith and Illingworth (1998) is adopted to have an independent estimate of the path integrated differential attenuation. Given the relationship between \( \alpha_0 \) and \( \alpha_{dp} \), it also represents a validation for the estimation of PIA. The aforementioned approach is based on the assumption that, in the stratiform
tail of the rain cell (where $Z_h < 20\text{dBZ}$), $Z_{dr}$ is expected to be close to zero given the predominant spherical shape of raindrops.

Starting from this assumption, $PIDA$ can be estimated as difference between the observed and expected values of $Z_{dr}$. As suggested in Bringi et al. (2001), for higher values of $Z_h$ ($20 < Z_h < 40 \text{dBZ}$) the expected values of $Z_{dr}$ can be estimated as a function of $Z_h$. For this purpose, a basic correction of $Z_h$ is initially performed by applying (4) using a standard value of $\gamma_h$. In order to minimize the impact of noise, an average estimate of the “true” $PIDA'$ has been performed by considering a range path of at least 20 gates. Figure 1 shows the observed $Z_{dr}$ during the event of 23 June 2005 at 15:45 UTC. The effects of attenuation are clearly visible resulting in high azimuthal heterogeneity of the $Z_{dr}$ PPI. This effect is strongly mitigated by all the examined correction techniques (see Figure 2). Figure 3 shows the histogram of the error ($\varepsilon_{PIDA} = A_{dr}(r_z) - PIDA'$) obtained estimating the $PIDA$ by the various attenuation correction procedures relatively to the whole event of 23 June 2005. The results, summarized in Table 1, show that the OPDP method performs relatively better in terms of mean error, the error standard deviation being similar to those obtained using the other methods.

![Figure 1](image1.png)

**Figure 1** PPI of $Z_{dr}$ observed on 23 June 2005 at 15:45 UTC.

![Figure 2](image2.png)

**Figure 2** PPI of $Z_{dr}$ after attenuation compensation. The upper, middle and lower panels refer to OPDP, FV and ZPHI, respectively. Event of 23 June 2005 at 15:45 UTC.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>$\bar{\varepsilon}_{PIDA}$</th>
<th>$\sigma_{PIDA}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>OPDP</td>
<td>-0.331</td>
<td>0.786</td>
</tr>
<tr>
<td>FV</td>
<td>-0.634</td>
<td>0.752</td>
</tr>
<tr>
<td>ZPHI</td>
<td>-0.852</td>
<td>0.837</td>
</tr>
</tbody>
</table>

Table 1. Numerical results obtained estimating the PIDA expressed in terms of mean error and error standard deviation. The results refer to the whole event of 23 June 2005.
4 Conclusions

As outlined by several authors, rain path attenuation schemes making use of differential phase shift may be in error due to temperature, shape and size distribution dependency. An optimized version of the standard $\Phi_{dp}$-based method for attenuation correction is evaluated in this work. A preliminary classification of the prevailing rain regime is accomplished adopting the maximum a posteriori probability criterion by means of the estimated $K_{dp}$, which is immune to calibration error and attenuation. The identified rain condition enable the use of appropriate relationships between specific attenuation and specific differential phase. A comparison with other advanced polarimetric techniques is performed on a large number of events observed in the Paris area by the dual-polarized radar operating in Trappes. An independent estimate of the path integrated differential attenuation is used as a reference to evaluate the various correction techniques. Numerical results have shown that the proposed methodology performs slightly better than the other examined techniques.

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References