

# Measurements of raindrop-size distributions from dual-polarization spectral observations

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## 1 Introduction

Rainfall radar observations depend on underlying raindrop-size distributions. Since rain rates retrieval from radar measurements is problem of current interest there is great importance in independent measurements of naturally occurring drop-size distributions. Often in situ measurements of raindrop size distributions are performed using disdrometers (Bringi et al. 2003). Despite wide acceptance of these measurements as ground truth, small sampling volume limits the accuracy of these measurements.

The relation between raindrop fall velocity and equivolumetric diameter (Atlas et al. 1973) enabled the possibility of converting vertical Doppler radar measurements to raindrop size distributions. This approach allows for increasing observation volume to radar resolution volume scales. Since Doppler power spectra of rainfall are influenced by turbulence and wind, the DSD retrieval procedure is not straightforward. Atlas et al. (1973) have shown that if effects of turbulence and wind are not accounted for the retrieved DSD would be largely erroneous.

Parameterization of raindrop size distributions (Ulbrich 1983) allowed for using optimization approach to find parameters of DSD. Hauser and Amayenc (1981) have shown that under certain assumptions intercept parameter and median volume diameter of exponential DSD can be retrieved from Doppler spectra observations. Their proposed method, however, did not include effects of turbulence and wind. Williams (2002) have extended this approach by including spectrum broadening and wind effects and assuming that raindrop-size distributions follow gamma functional form that allowed DSD parameters retrieval from profiler measurements. Similar procedure was also implemented by Moisseev et al. (2006) for scanning radar measurements. The main limitation of these methods,

though, is dependence on one or another functional form of the DSD.

Recently (Moisseev et al. 2006; Spek et al. 2005; Unal et al. 2000) have combined dual-polarization and to retrieve microphysical properties of precipitation. Moisseev et al. (2006) have shown that by taking radar measurements at high elevation angles, between 30 and 60 degrees, one can successfully combine Doppler observations with dual-polarization measurements for precipitation studies. Furthermore, it was shown that spectral differential reflectivity measurements can be related to raindrop shapes and air motion. In this work we propose to use spectral differential reflectivity observations to estimate spectrum broadening kernel width and wind velocity. Then by applying this information one can remove influence of spectral broadening and wind from a Doppler power spectrum and therefore retrieve raindrop-size distribution.

To validate the proposed technique, sensitivity of this method to the underlying assumptions and calibration errors is evaluated on simulated of radar measurements. Application of the method to CSU-CHILL measurements of stratiform precipitation is presented.

## 2 Methodology

If radar measurements are taken with elevation angle ranging between 30 and 60 degrees, one can combine dual-polarization observations with Doppler spectra observations to get more insight into precipitation microphysics (Moisseev et al 2006). In case of no spectrum broadening and no wind the spectral differential reflectivity can be defined as:

$$Z_{dr}(v) = \frac{S_{pr}^{hh}(v)}{S_{pr}^{vv}(v)} = \frac{\sigma^{hh}(D)}{\sigma^{vv}(D)} \quad (1).$$

One can see that in this particular case spectral differential reflectivity depends only on two co-pol scattering cross-sections. This ratio is fully defined by raindrop shapes and

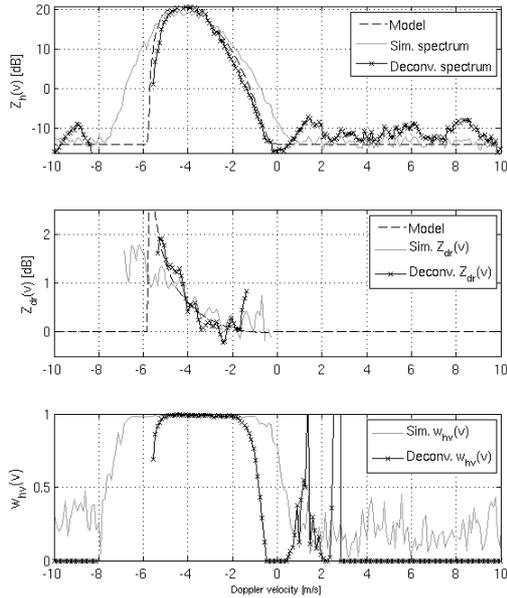


Figure 1. Illustration of the method performance on the simulated data. The top figure shows power spectra before and after deconvolution, represented by grey and solid line with x- markers respectively. Also for comparison purposes the model spectra is shown by the dashed line. The middle figure shows spectral differential reflectivity before and after deconvolution. And the bottom figure shows corresponding coherency spectra.

therefore can be related to an existing raindrop size-shape relation. There is a number of known relations that either based on experimental observations (Andsager et al. 1999; Pruppacher and Beard 1970; Chandrasekar et al. 1988; Bringi et al. 1998) or theoretical considerations (Beard and Chuang 1987).

In order to retrieve drop-size distributions from Doppler spectra measurements, one needs to compensate for the effect of spectrum broadening and wind. Gossard (1988) and Rajopadhyaya et al. (1993) have shown that VHF profiler measurements of clear air and precipitation returns can be used to estimate raindrop-size distributions. The windprofiling DSD retrieval procedure is based on deconvolution of observed precipitation spectrum by using clear-air signal spectrum as the broadening kernel (Gossard (1988); Rajopadhyaya et al. (1993)). This method gives a direct way of retrieving raindrop-size distributions from VHF Doppler radar observations. For shorter wavelength radars scattering from hydrometers is much larger than Bragg scattering, and therefore this method is not applicable.

Let us consider dual-polarization slant profile spectral measurements taken at an elevation angle ranging between 30 and 60 degrees. It was discussed that in the absence of

spectral broadening and wind the spectral differential reflectivity is completely defined by a drop size-shape relation. Assuming that this relation is known, one can use expression (1) as the reference differential reflectivity spectrum that corresponds to Doppler power spectra due to

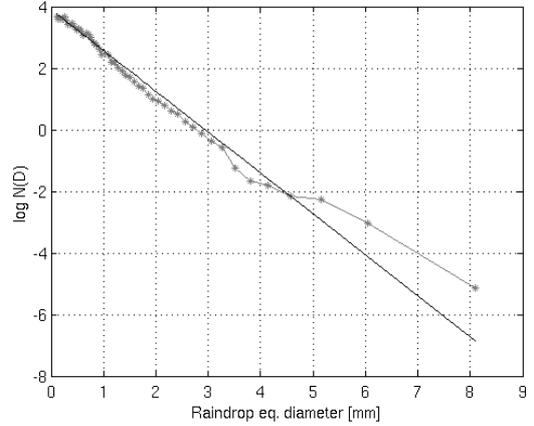


Figure 2. An example of the retrieved and input drop-size distribution. The retrieved DSD is depicted by grey line with star markers. And the input DSD is shown by solid lines. Retrieval is based on the simulated spectrum shown in Fig. 1.

scattering from hydrometeors. Furthermore the expression (1) is independent of DSD. Therefore estimation of the spectrum broadening kernel width and wind radial velocity component can be formulated as a least square problem of finding parameters  $\sigma_b$  and  $v_0$  that minimize the sum square residuals, SSR:

$$SSR = \sum \left( Z_{dr}^{dec}(v) - \frac{\sigma^{hh}(D)}{\sigma^{vv}(D)} \right)^2. \quad (2)$$

Here  $Z_{dr}^{dec}(v)$  is obtained by applying deconvolution to the observed  $hh$  and  $vv$  power spectra with convolution kernel  $S_b(v - v_0, \sigma_b)$ .

To illustrate feasibility of the procedure we have simulated dual-polarization radar observations using method described by Chandrasekar et al. (1986). For these simulations shape of spectra was assumed to be determined by the drop-size distribution, which follows gamma functional form, and convolution kernel that follows Gaussian shape. For each simulation run Doppler spectra were estimated by averaging over 30 simulated spectra.

The DSD retrieval procedure consists of three steps. At the first step parameters of the convolution kernel are found. As described above this step is carried out by solving the following minimization problem:

$$\min_{v_0, \sigma_b} \sum \left( Z_{dr}^{dec}(v) \Big|_{v_0, \sigma_b} - \frac{\sigma^{hh}(D)}{\sigma^{vv}(D)} \right)^2. \quad (3)$$

For deconvolving Doppler spectra we have used iterative technique proposed by Lucy (1974). To minimize effect of noise on deconvolved spectra weights depending on values of co-polar coherency spectrum were assigned to spectral lines.

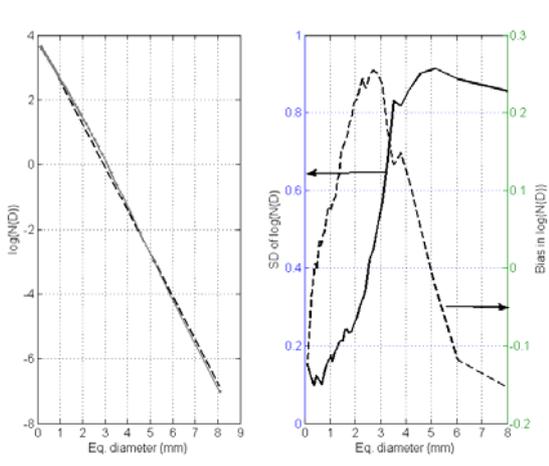


Figure 3. Effect of assumed raindrop size-shape relation on the retrieval. The left figure shows retrieved and input DSDs, solid and dashed lines respectively. The retrieved DSD is obtained by averaging resulting DSD from 100 simulations. The right figure shows the bias and standard deviation in the retrieved  $\log N(D)$ .

At the second step the convolution kernel is used to recover  $S_{pr}(v)$ . In the Fig. 1 simulated and recovered power spectra, spectral differential reflectivity and coherency spectra are shown. It can be seen that recovered spectrum, solid line with crosses, closely follows modeled spectrum where  $\sigma_b = 0$ . It is interesting to note that deconvolution procedure did not affect much the coherency spectrum values in the precipitation region. In Fig. 2 retrieved DSD is shown.

### 3 Evaluation of the methodology

#### 3.1 Effect of drop size-shape relation

In the previous section we have shown that proposed method is able to retrieve DSD under assumption that raindrop size-shape relation is known. Currently there is a number of known relations that would result in different  $Z_{dr}(v)$  values especially for diameters ranging between 1 and 4 mm. In this diameter range relations by Pruppacher and Beard (1970) and by Andsager et al. (1999) depart the most. Therefore, to study dependence of the proposed method on the raindrop shape assumption we have simulated 100 spectra using Andsager et al. (1999) size-shape relation. Then the spectrum broadening kernel width and wind velocity radial component were retrieved, where the reference spectral differential reflectivity (1) was calculated assuming (Pruppacher and Beard, 1970) relation. The results of this study are given in Fig. 3. It can be seen that as a result of the

uncertainty in the drop size-shape relation, one might expect increase in the bias of the estimate for diameters less than 5 mm. Nonetheless, the resulting bias is still less than 0.2. The standard deviation of the estimate remains the same and does not depend on the drop shape assumption.

#### 3.2 Effect of $Z_{dr}$ calibration

In the proposed method the retrieval of the spectrum broadening kernel width and wind velocity depends on  $Z_{dr}(v)$  observations and hence can be expected to be influenced by the calibration errors. To investigate this effect we have simulated three datasets of 100 spectra each, with  $Z_{dr}$  offsets of 0.1, 0.2 and 0.5 dB. Then the DSD retrieval method was applied to each dataset. The results of this study are summarized in Fig. 4. It can be observed that  $Z_{dr}$  offset has a strong influence on the bias. It ranges between 0 and 0.5 for the offset of 0.1 dB, -0.2 and 1 for the offset of 0.2 dB and between -1 and 1.7 for the offset of 0.5 dB.

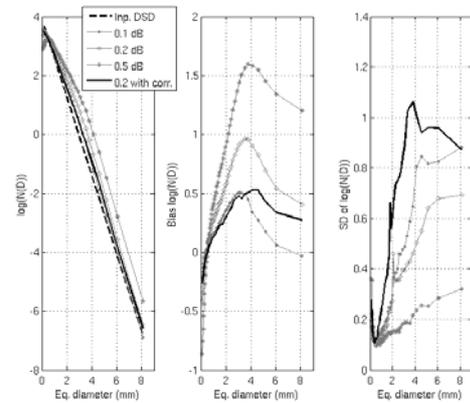


Figure 4. Influence of  $Z_{dr}$  bias on the retrieval. The left figure shows retrieved and input DSD. The middle figure shows the biases in  $\log N(D)$  for different  $Z_{dr}$  offsets. The right figure shows the standard deviations in the retrieved  $\log N(D)$ .

Another way of approaching this problem would be to consider  $Z_{dr}$  offset as an additional unknown parameter and to solve (3) for three parameters,  $\sigma_b, v_0$  and the offset. This approach was applied to the simulated radar observations with  $Z_{dr}$  offset of 0.2 dB. The resulting DSD, bias and standard deviation are shown in Fig. 5 as black solid lines. It can be seen that this approach reduces bias in the DSD estimate that as a result ranges between -0.25 and 0.5, but it also increases the standard deviation of the estimate.

### 4 CSU-CHILL data

On July 23, 2004 time series data measurements were collected during stratiform rain event by the CSU-CHILL radar. The observed reflectivities for this event were around 35 dBZ. To investigate performance of the DSD retrieval method, the observed spectra for each range gate were averaged over complete dataset. Prior to averaging all spectra were shifted to the same mean velocity (Moissev et

al. 2006). Then the method was applied to the averaged

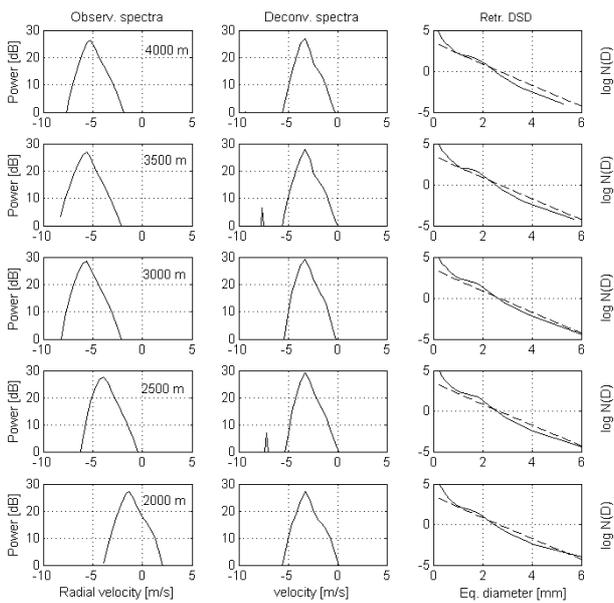


Figure 5. Retrieved DSD from CSU-CHILL measurements. Figures in the left column show observed Doppler spectra at different ranges. The middle column presents deconvolved spectra for given ranges. The figures in the right column show retrieved DSD, as depicted by solid lines. Moisseev et al. 2006 have shown that for this dataset  $\log N_w = 3.54$ ,  $D_0 = 1.2$  and  $\mu = -0.4$ , as shown by dashed lines.

spectra. For the estimation of the convolution kernel, Beard and Chuang (1987) raindrop shapes were assumed. The retrieved DSD were further averaged over 5 neighboring range gates, resulting in the effective range resolution of 250 m. The resulting raindrop-size distributions for select range gates are shown in Fig. 5. Moisseev et al (2006) have used the same dataset to estimate precipitation DSD parameters by fitting parametric model to the observed spectra. It was concluded that this measurement can be characterized by a gamma DSD with  $\log N_w = 3.54$ ,  $D_0 = 1.2$  and  $\mu = -0.4$ . The resulting  $\log N(D)$  plot is shown in the Fig. 5. One can observe that DSD retrievals from these two methods are in good agreement.

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