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Rain estimation from partially filled scattering volumes

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1 Introduction

Atmospheric parameters retrieved by remote sensing techniques often represent averages over considerable space or time intervals. Unresolved variability of corresponding primary physical values is a problem as soon as non-linear relations between the primary physical value and the retrieved parameter come into play, as for example the *R*(*Z*)-relation between radar reflectivity *Z* and rain rate *R*. While this is recognized to be a crucial issue for many space based techniques (particularly for cloud and rainfall retrievals) due to the large sensor footprints (Durden et al. 1998, Heymsfield et al. 2000), ground based radar measurements of rainfall are sometimes considered as a reference having sufficient resolution with negligible unresolved variability. On the other hand precipitation is known to show strong variability even on very small scales. There is in fact evidence that no scale exists for spatial hydrometeor distributions, where Poisson statistics would be an adequate description analogue to molecules of a gas in thermodynamic equilibrium (Jameson, 2005). Radar calibration using distrometer-data is therefore faced with the problem of non-matched spatial resolution of distrometer- and radar-data respectively. If radar rain fall is adjusted with rain gauge networks, one may argue, that such effects are included already in the adjustment. But this argument would hold only, if the pertinent spatial distribution parameters would be universal for all rain events. In addition, the spatial resolution of radar data (at least of raw data) is range dependent.

Data, obtained with a fine resolving local-area X-band radar (WRDR) (Peters et al., 2006) during the LAUNCH2005 campaign (www.dwd.de/en/FundE/Observator/MOL/MOL.htm), provided the opportunity for a new look on the relevance of unresolved variability for rain fall retrievals using standard weather radars. The main system and operating parameters of the WRDR are given in Peters et. al. (2006). WRDR reflectivities, recorded with a primary resolution of 60 m × 2°, were used to investigate the impact of unresolved variability in standard weather radar signals which were simulated by averaging the data over 4 to 5 kilometer range and 12° azimuth sector corresponding to about 1 km^2 horizontal resolution.

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2 Inhomogeneity of rain drop density

Due to the stochastic position of hydrometeors "inhomogeneity" of drop density and accordingly of the radar reflectivity must be defined in a statistical sense. The single shot radar reflectivity of an ensemble of hydrometeors Z is a random variable with an exponential probability density function (pdf) (e.g. Doviak and Zrnic, 1993):

$$p(Z) = \frac{1}{\sigma}\exp(-Z/\sigma) \tag{1}$$

which has the well-known property,

$$\bar{q} \equiv \frac{\sigma}{\bar{Z}} = 1 \tag{2}$$

where $\bar{Z}$ is the expected value ($\bar{Z} = \int_0^\infty p(Z)ZdZ$), and σ is the standard deviation ($\sigma^2 = \int_0^\infty p(Z)(Z-\bar{Z})^2$.

Estimates of $\bar{Z}$ and σ (indicated by Z_e and σ_e respectively) can be obtained by sampling Z in n different volumes calculating:

$$Z_e = \frac{1}{n}\sum_{i=1}^{n} Z_i \tag{3a}$$

$$\sigma_e = \mathrm{sqrt}\left(\frac{1}{n-1}\sum_{i=1}^{n}(Z_i - Z_e)^2\right) \tag{3b}$$

We introduce the "inhomogeneity-parameter"

$$q_e \equiv \frac{\sigma_e}{Z_e} \tag{4}$$

which exhibits some random fluctuations due to the finite sampling size n. But only if Z follows the same pdf in all sampled volumes, i.e. in the statistical homogeneous case, the expectation value $\overline{q}_e$ is unity according to Eq. 2. In all other cases $\overline{q}_e$ will exceed unity, and its value can be used as measure for the statistical inhomogeneity.

The pdf of Eq. 1 is a rather common feature of incoherent wave fields, resulting of the superposition of many stochastically distributed phasors. Therefore this pdf holds also true for the receiver noise power, which remains, if there are no atmospheric targets. (The receiver noise power as well as the signal power is presented here by its corresponding radar reflectivity). In contrast with the scattered signal from precipitation one may assume that the receiver noise is stationary which results in $\overline{q}_{\mathrm{e,\,noise}} = 1$.

3 The data set

The WRDR-sampling and -averaging scheme used for this analysis is illustrated in Fig.1. Only data from a ring between 4 and 5 km radius were used, in order to minimize artifacts due to potential range dependencies of the retrieved radar reflectivity. The pixels of the spatio/temporal reflectivity field represent 60 m range-, 2° azimuth-sector-, and 30 s time-intervals.

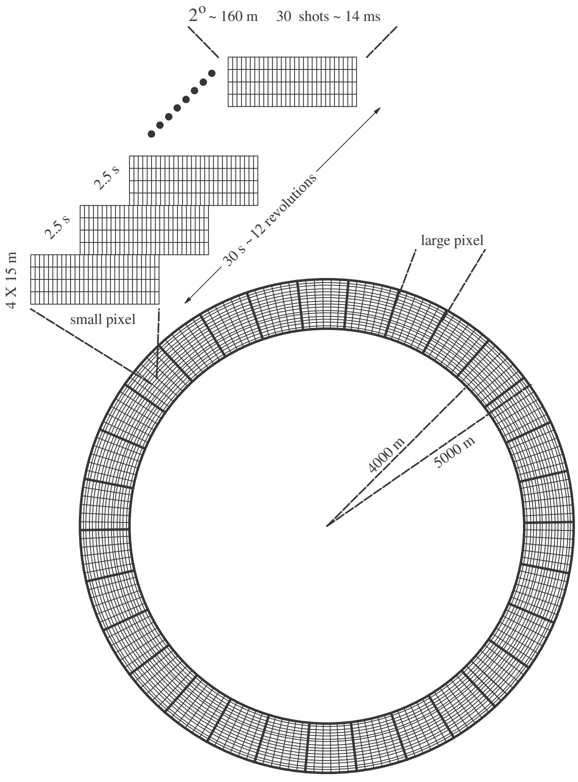

Fig.1. WRDR Sampling scheme. Small Pixel: 60 m × 160 m. Large Pixel: 960 m × 960 m (= 16 × 6 small pixels).

Not only the mean values Z_S but also the standard deviations σ_s of the single shot, single range gate reflectivities Z were recorded for each pixel. Each pixel consists of about 1400 samples: (4 range resolution cells of 15 m × about 30 shots per 2° azimuth increment × 12 antenna revolutions per 30 s averaging time). The number of *independent* samples n_f is considerable smaller, due to the antenna beamwidth of 2.5° and the finite coherence time of the signal. A lower limit is $n_f = 48$, which is equal to the product of range gates and antenna revolutions included in the small pixel.

In order to simulate the resolution of typical weather radar data (see section 5), groups of 96 pixels (henceforth called *small pixels*) were integrated to *large pixels* consisting of 16 *small pixels* in range and 6 *small pixels* in azimuth, corresponding approximately to squares of about 1 km^2 area.

4 Analysis of the *Z*-field inhomogeneity.

The small-pixel data were used to calculate

$$q_S \equiv \frac{\sigma_S}{Z_S} \tag{4}$$

In case of low signal to noise ratio the receiver noise is dominating, which is assumed to be stationary with the expectation value $\overline{q}_S = 1$, whereas effects of inhomogeneity should become visible in case of high signal to noise ratio. Consequently we selected time intervals containing fairly contiguous rain events and stratified these data according to Z_S.

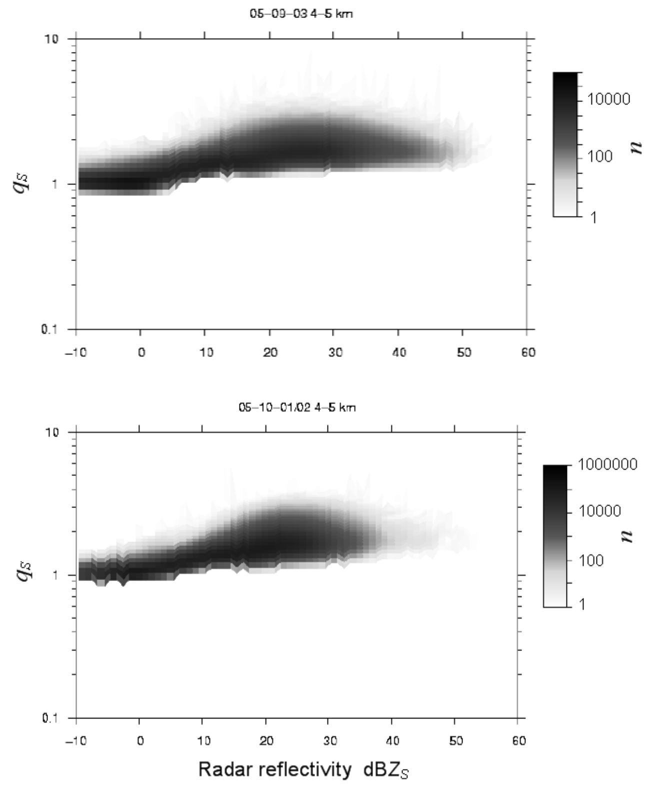

Fig.2. q_S versus radar reflectivity Z_S. The grayscale indicates the number n of samples corresponding to a q_S, Z_S -pair. Top: Convective rain, Bottom: Stratiform rain.

(Due to the limited range interval from 4 to 5 km there is a close relation between Z_S and signal to noise ratio.) The results are shown in Fig. 2 for two rain events, each comprising several hours. The first event is characterized by convective showers and low winds, while the second event is of stratiform type with a well defined melting layer and moderate to high winds. The regressions of q_S versus radar reflectivity are presented in 2-dimensional histograms, where the grayscale indicates the number n of samples corresponding to a given q_S, Z_S -pair. The mean values of radar reflectivity range from -10 to 50 dBZ_S during these periods. One recognizes that $\overline{q}_S$ is close to unity for radar reflectivities below 0 dBZ_S, which is the noise threshold of the WRDR at this range. At higher radar reflectivities $\overline{q}_S$ increases and reaches a saturation level of $\overline{q}_S \approx 2$, which demonstrates that inhomogeneity plays a role even on the scale of the *small pixels*. Very similar distributions were found for both rain events, although they were of different type. This observation supports the above mentioned statement of Jameson (2005) who pointed out that spatial inhomogeneity seems to be a general feature of precipitation at all scales, which is in fundamental contrast to the distribution of molecules of gas in thermodynamic equilibrium. Consequently there is no scale, which could be associated with the "right" $R(Z)$-relation. Furthermore, any specification of a $R(Z)$-relation is only sensible together with the associated scale.

5 Estimating sub-resolution inhomogeneity

Large pixel data (see Fig. 1) were obtained by calculating the arithmetic mean Z_l and the standard deviation σ_l of the linear *small pixel* radar reflectivities

$$Z_l = \frac{1}{l}\sum_{i=1}^{l} Z_{S,i} \tag{5a}$$

$$\sigma_l = \mathrm{sqrt}\left(\frac{1}{l-1}\sum_{i=1}^{l}\left(Z_{S,i} - Z_l\right)^2\right) \tag{5b}$$

The Z_l-data are considered to represent standard weather radar resolution. On the basis of Eq. 5a and 5b a *large pixel* inhomogeneity index

$$q_l = \frac{\sigma_l}{Z_l} \tag{6}$$

was established. In contrast with σ_S in section 4, σ_l is not based on the single-shot variability of Z rather than of the small pixel mean values Z_S. Therefore q_l should approach zero for homogeneous conditions within $\mathrm{sqrt}(1/n_f)$ uncertainty. The deviation of q_l from zero is tentatively used as a measure for the unresolved inhomogeneity within a large pixel.

In order to test the significance of unresolved inhomogeneities for weather radar data a standard $R(Z)$-relation of the form $Z = aR^b$ with $a = 250$ and $b = 1.6$ was applied to calculate the rain rate $R(Z_l)$ and alternatively $R_l(Z_S)$ with

$$R_l(Z_S) = \frac{1}{l}\sum_{i=1}^{l} R(Z_{S,i}). \tag{6}$$

While $R(Z_l)$ is considered to represent standard weather radar results with 1 km^2 horizontal resolution, $R_l(Z_S)$ is the mean rain rate over 1 km^2 area as obtained from rain retrievals in all sub-resolution cells of about 10^{-2} km^2 area. In case of a linear $R(Z)$-relation the order of averaging would not matter and the identity $R(Z_l) \equiv R_l(Z_S)$ would hold true. But the exponent $b = 1.6$ will give rise to differences between $R(Z_l)$ and $R_l(Z_S)$ as soon as inhomogeneities are present. As discussed in section 4, we do not claim that $R_l(Z_S)$ is closer to truth than $R(Z_l)$, nevertheless the ratio $q_R = R(Z_l)/R_l(Z_S)$ indicates, whether there is a scale dependence of $R(Z)$ or not.

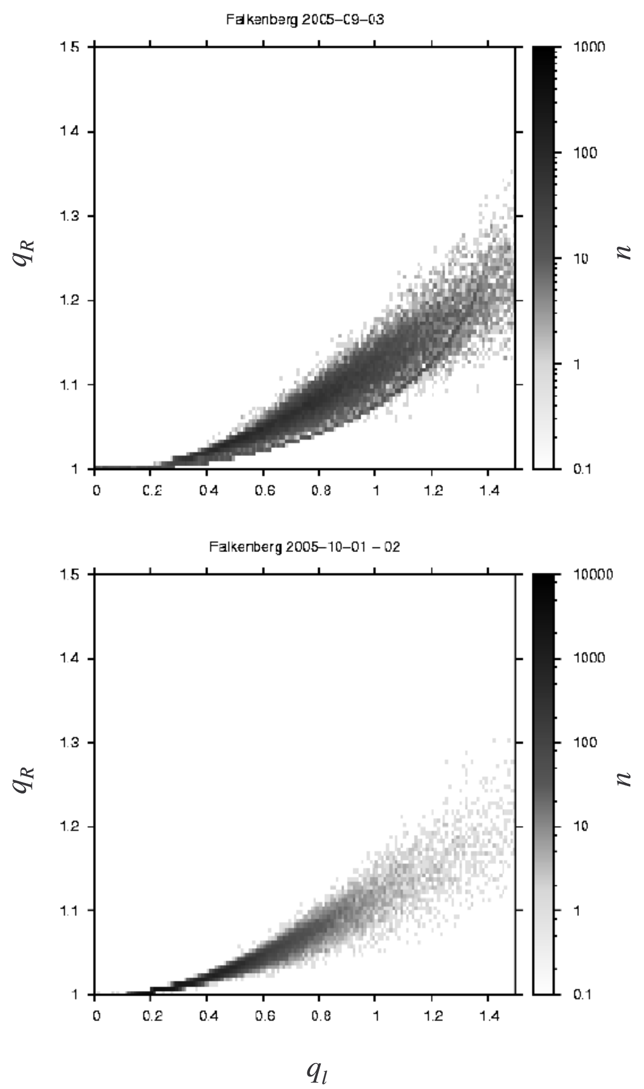

Fig.3: q_R versus q_l. The grayscale indicates the number of samples corresponding to a given q_R, q_l – pair. Top: Convective. Bottom: Stratiform.

In order to visualize this scale dependence and to test, if q_l is a sensible measure for the effect of inhomogeneities, q_R is plotted versus q_l in Fig. 3 for the same rain events as in Fig.2. Again two-dimensional grayscale histograms were chosen to present the regression.

The shape of the regression between q_R and q_l seems to be fairly universal. In both cases one recognizes that the scale-dependence q_R assumes maximum values of 1.3, which occur, as expected, at the highest values of q_l. The scatter of the regressions is explained by the fact that the second moment q_l is not a full description of the pdf of Z_S. The width of the scatter appears to increase in proportion to $q_R - 1$ suggesting a relative uncertainty of some ±30% for the estimate of q_R, which would be a significant improvement over an a-priori estimate. Unfortunately, the determination of q_l requires sub-resolution information, which is not available in standard radar data.

In order to derive an inhomogeneity index from resolved radar data, we invoked the property of scale free fields (as for example turbulence in the inertial subrange), which allows inferring the variance in a certain scale regime from the measured variance in another scale regime. In case of scale freedom of the radar reflectivity field, it should be therefore possible to infer the sub-resolution variance (and hence q_R) from the variance of resolved values Z_l. This could be estimated from Z_l measurements in the environment of the radar volume under consideration. In a first attempt $r = 10$ successive Z_l-values, measured during a 5 min interval centered around the time of consideration where used to calculate a large scale variance

$$\sigma_{xxl} = \mathrm{sqrt}\left(\frac{1}{r-1}\sum_{i=1}^{r}\left(Z_l - Z_{xxl}\right)^2\right) \qquad 7$$

with Z_{xxl} = 5 min mean value of Z_l. In analogy to q_l a large scale inhomogeneity index q_{xxl} was introduced:

$$q_{xxl} = \sigma_{xxl} / Z_{xxl} \qquad 8$$

The regression of q_r versus q_{xxl} is shown in Fig.4 for the stratiform rain event. No improvement over a-priori estimates is achieved by stratifying the data according q_{xxl}. The same applies to the convective event (not shown here).

6 Summary and outlook

Inhomogeneity has been shown to be a ubiquitous property of radar reflectivity fields. Therefore the non-linear $R(Z)$-relation is generally scale dependent. Transition from *small pixels* (60 m ×160 m) to *large pixels* (960 m × 960 m) causes relative overestimation of rain rate up to 1.3 according to observations of convective and stratiform rain events. The scale dependence is not a fixed function but varies with the actual inhomogeneity.

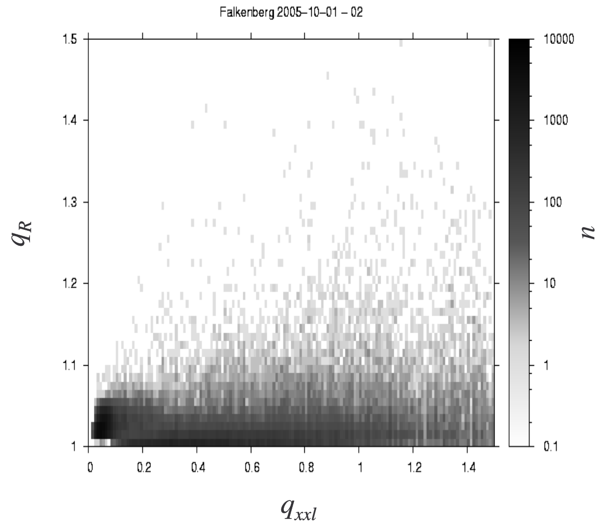

Fig.4: Stratification of the scale dependence with the Z_l-variance q_{xxl} derived in the 5 min environment of the time of consideration in Fig.1.

A first attempt to infer the scale dependence q_R from standard weather radar data without sub-resolution information was not successful. The time domain averaging was a convenient but certainly not optimum choice for the environment of the volume under consideration. A different shaping of the environment, including all dimensions of the radar reflectivity field, would allow collecting larger sample sizes than just sampling in the time domain. On-going studies will show, if this could improve the prediction score of q_{xxl}.

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