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# Enhancement of Rain Retrieval from Radar Profilers

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## 1 Introduction

Since many years Doppler radar profilers are known as valuable tools to retrieve profiles of rain rate and further rain parameters (e.g. Wakasugi et al. (1986)). Applying well established relations for terminal fall velocity, shape and backscatter cross section of rain drops allows to derive rain rate without empirical assumptions. For calculating the backscatter cross section, the flattening of rain drops (see e.g. Pruppacher and Klett (1997)) constitutes a non-negligible error source.

In this contribution, four different algorithms are applied to calculate the backscatter cross section of rain drops. Their results are compared to each other. A point matching algorithm serves as reference. To this reference, results from a Mie scattering algorithm, the Rayleigh approximation and an algorithm correcting the backscatter cross section according to the Rayleigh approximation for drop flattening are compared. The findings from this inter-comparison are presented for transmitting frequencies of 10 GHz, 24 GHz and 94 GHz.

## 2 Motivation

Calculating the backscatter cross section from raindrops with a diameter of more than 1 mm constitutes a challenge at transmitting frequencies above 10 GHz. At these frequencies, the backscatter from larger raindrops must not be calculated according to the Rayleigh approximation any more. Only point matching algorithms or calculations applying the T-matrix method provide reliable results for raindrops of diameters larger than 1 mm, which significantly deviate from spherical shape due to drop flattening.

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In the presence of flattened drops, neither the Rayleigh approximation nor the Mie method (which is exact for spherical targets) provide the correct results. For radar profilers operating at frequencies below 10 GHz, algorithms to correct the backscatter cross section calculated by the Rayleigh approximation have been published (e.g. Klugmann and Richter (1992)). For radar profilers operating at higher frequencies, point matching algorithms deliver reliable results, but require high computing time.

Hence, methods or algorithms to calculate backscatter cross sections from flattened drops faster are desirable. This could be an algorithm based on the Mie scattering applying a drop diameter dependent correction factor. However, other options should be investigated as well.

## 3 Theory

The results from different scattering algorithms are compared. A point matching code for flattened drops according to Oguchi serves as reference. Its results are compared to these from a Mie scattering code and the Rayleigh approximation for spherical drops, and from a code for Rayleigh scattering from flattened drops. The codes for flattened drops assume the same axis asymmetry relation $\beta$ of the drop main axes $b$ (larger axis) and $a$. We used the drop flattening described by the relation given by Oguchi (1973)

$$\beta = \frac{b}{a} = \frac{1}{1 - C_{ob} \cdot D} \qquad \left( C_{ob} = \frac{0.41}{4.5} \right), \qquad (1)$$

with $D = (a \cdot b^2)^{1/3}$ describing the equivolumic drop diameter in [mm]. For drops of spherical shape, the equivolumic diameter is the same as the diameter of the sphere.

According to well established knowledge, it is assumed that rain drops exist in a stable state in the range of $D \leq 6$mm. Drops of larger diameter may occur for a short time, e.g due to drop collision, but will decay very fast. Hence we restrict our calculations to drops of $D \leq 7$mm.

Including the transmitting frequency of 94 GHz into this study is motivated by the fact, that a growing number of cloud profilers are operated at this frequency. These systems also can be used to investigate rain in and below precipitating clouds. They offer the additional benefit, that the oscillations of the backscatter cross section in the Mie region provide valuable information about the location of various drop diameters in the backscatter spectrum. Using the minima of the backscatter spectra to localize the corresponding drop diameter and terminal fall velocity, the vertical wind speed can be accessed with good accuracy.

### 3.1 Point matching method

Scattering calculations from a point matching code for flattened drops (spheroids) according to Oguchi (1973) serve as reference for the other codes investigated in this contribution. The point matching method is suitable to calculate the scattering function from arbitrary target shapes, e.g. flattened drops.

### 3.2 Mie scattering

The Mie scattering theory was developed and published by Mie (1908). The complex backward scattering function *f* as defined by the relation (*E* is the field strength)

$$\hat{E}^s = \hat{E}^i f \frac{e^{-jkr}}{r}, \qquad (2)$$

with $j = (-1)^{1/2}$ and $r$ describing the distance between observation point and target, is given by

$$f = \frac{-j\lambda^3}{\pi^3 D^2}\left[\sum_{n-1}^{\infty}(-1)^{(n+1)}(2n+1)(a_n - b_n)\right]^*, \qquad (3)$$

where λ denotes the vacuum wavelength of the electro-magnetic radiation and D the diameter of the spherical drops. The coefficients $a_n$ and $b_n$ according to Mie (1908) depend on the complex relative refractivity $\varepsilon_r = \varepsilon / \varepsilon_0$ of the material and on the diameter $D$ of the scattering sphere. The backscatter cross section σ then is calculated by the relation

$$\sigma_M = 4\pi |f|^2. \qquad (4)$$

The Mie backscatter calculations presented in this contribution were performed using a program coded and published by Fiser (1993). This code, originally written in BASIC, is now ported to Matlab.

### 3.3 Rayleigh scattering from flattened drops

The Rayleigh scattering theory was developed and published by Strutt (1871) – later Lord Rayleigh. The theory gives an approximation for the scattering of electro-magnetic radiation from spheres. It is valid for sphere diameters significantly smaller than the vacuum wavelength of the radiation. The backscatter cross section is given by

$$\sigma_R = \frac{\pi^5}{\lambda^4}\left|\frac{(\varepsilon_r - 1)}{(\varepsilon_r + 2)}\right|^2 D^6 \qquad \left(\frac{\pi \cdot D}{\lambda} << 1\right), \qquad (5)$$

where $\varepsilon_r$ denotes the complex relative permittivity of the target medium. The Rayleigh approximation assumes spherical drop shape.

The flattening of rain drops constitutes a non-negligible error source. The Rayleigh scattering from flattened drops was calculated by a code according to Klugmann and Richter (1992). The correct backscatter cross section $\sigma_f$ is achieved by dividing the Rayleigh backscatter cross section $\sigma_R$ through a correction factor κ:

$$\sigma_f(D) = \frac{\sigma_R(D)}{\kappa(D)}. \qquad (6)$$

Besides from the equivolumic drop diameter $D$, this correction factor κ depends on the axis ratio $\beta$ according to Eq. (1) and on the complex relative permittivity $\varepsilon_r$ of the scattering target material.

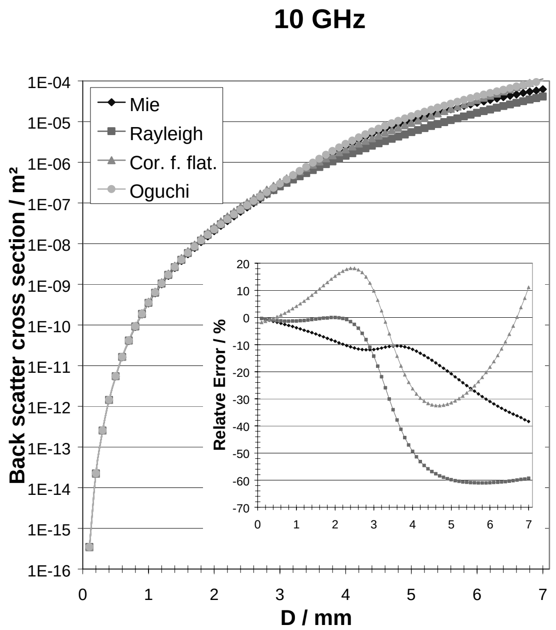


**Fig. 1.** Comparison of backscatter cross section versus drop diameter at 10 GHz calculated from the different scattering theories. The correction of the Rayleigh backscatter cross section for drop flattening is calculated according to Eq. (6). The small inset shows the relative error with respect to the backscatter cross section according to Oguchi, as defined by Eq. (7).

## 4 Results

The backscatter cross section of rain drops has been calculated using the methods described above for a temperature 5°C at transmitting frequencies of 10 GHz, 24 GHz and 94 GHz. The diameter range 0.1mm ≤ $D$ ≤ 7mm was covered by the calculations. The results of the inter-comparison between the different methods are displayed in Figs. 1 to 3. The main figures show the backscatter cross section versus drop diameter or – if flattened drop shape is assumed – equivolumic diameter. The small insets of these figures show the relative error of the backscatter cross sections calculated by the different methods with respect to the result of the algorithm according to Oguchi. The relative error is given by the formula

$$E_{rel}(D) = 100 \cdot \frac{\sigma_x(D) - \sigma_O(D)}{\sigma_O(D)} \ [\%], \qquad (7)$$

where $\sigma_O$ denotes the backscatter cross section according to Oguchi (1973), while the index $x$ in $\sigma_x$ can become $M$ (Mie backscatter cross section), $R$ (Rayleigh backscatter cross section) or $f$ (backscatter cross section according to Rayleigh approximation with correction for drop flattening).

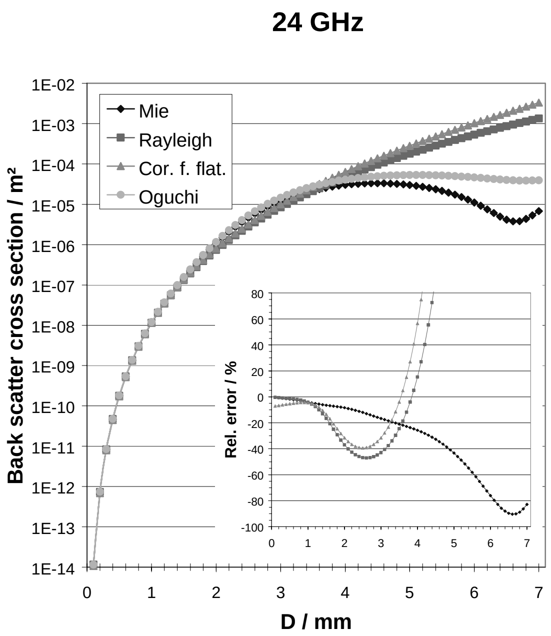


**Fig. 2.** Comparison of backscatter cross section versus drop diameter at 24 GHz calculated from the different scattering theories. The correction of the Rayleigh backscatter cross section for drop flattening is calculated according to Eq. (6). The small inset shows the relative error with respect to the backscatter cross section according to Oguchi, as defined by Eq. (7).

For Rayleigh scattering approximation the deviations from the Oguchi code results as expected become significant at higher drop diameters. But at 10 GHz the Rayleigh approximation shows the smallest deviation of all methods from the Oguchi values for $D \leq 3$mm. As one can see from Fig. 1, the relative error is even below 5% for $D \leq 2.5$mm. This error is in the order of magnitude of other errors that affect the accuracy of rain parameter retrieval. At 24 GHz and 94 GHz, the Rayleigh approximation only can be applied for $D \leq 1$mm.

Correcting the Rayleigh backscatter cross section for drop flattening shows the highest benefits at 10 GHz. As it can be seen from Fig. 1, the relative error is restricted to -30% / +20% in the whole diameter range. Nevertheless, the error induced to rain parameter retrieval by applying this method to calculate backscatter cross section is significant. At 24 GHz, the relative error is within ±40% for $D \leq 3.7$ mm. This can be read from Fig. 2. The correction for drop flattening does not very improve the applicability of the Rayleigh approximation for calculating the backscatter cross section.

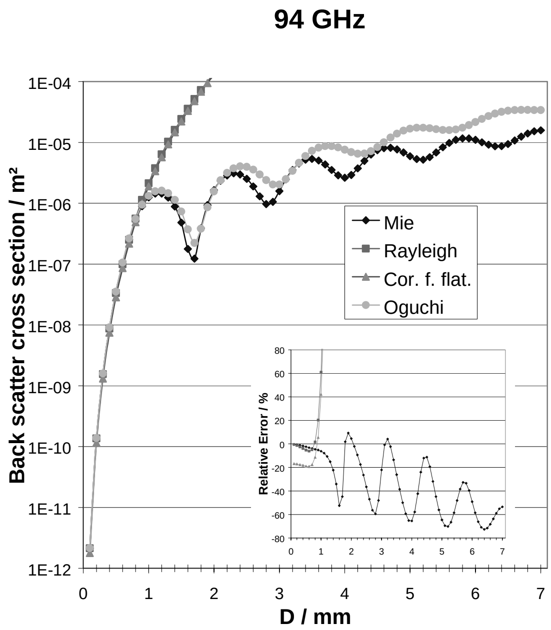


**Fig. 3.** Comparison of backscatter cross section versus drop diameter at 94 GHz calculated from the different scattering theories. The correction of the Rayleigh backscatter cross section for drop flattening is calculated according to Eq. (6). The small inset shows the relative error with respect to the backscatter cross section according to Oguchi, as defined by Eq. (7).

The Mie code for spherical drops shows deviations from the Oguchi code results in the values of the backscattering cross section as soon as the non-spherical character of the flattened drops becomes relevant. At 10 GHz (see Fig. 1), calculation of the backscatter cross section with the Mie algorithm does not give a significant advantage compared to calculation by the Rayleigh approximation with correction for drop flattening. At 24 GHz (Fig. 2) and 94 GHz (Fig. 3), the Mie backscatter cross section for $D > 1$mm is closest to the values calculated by the Oguchi algorithm. Nevertheless the relative error, which is negative virtually in the whole drop range for every investigated frequency, is up to -90 % and -70%, respectively.

Furthermore it is obvious especially from Fig. 3, that the location of the local minima and maxima of the Mie backscatter cross section increasingly differs in diameter with growing drop size and hence growing value of $\beta$. Numerical problems as the origin of these deviations can be excluded due to the excellent match between results from the Mie code and the Oguchi code applied for spherical drops. This behavior gave rise to the idea to correct the diameter $D$ for the Mie calculations into the scatter effective diameter $D_{se}$ by the formula

$$D_{se} = D \cdot \beta^{1/6}, \qquad (8)$$

where $\beta$ is the axis asymmetry relation defined by Eq. (1).

The result of this correction for a transmitting frequency of 94 GHz and a temperature of 5°C is displayed in Fig. 4. It can be seen that the location of the maxima and minima matches significantly better in Fig. 4 than for the non-corrected case displayed in Fig. 3. Nevertheless the value of the backscatter cross section calculated by the two different methods increasingly deviates from each other with increasing diameter.

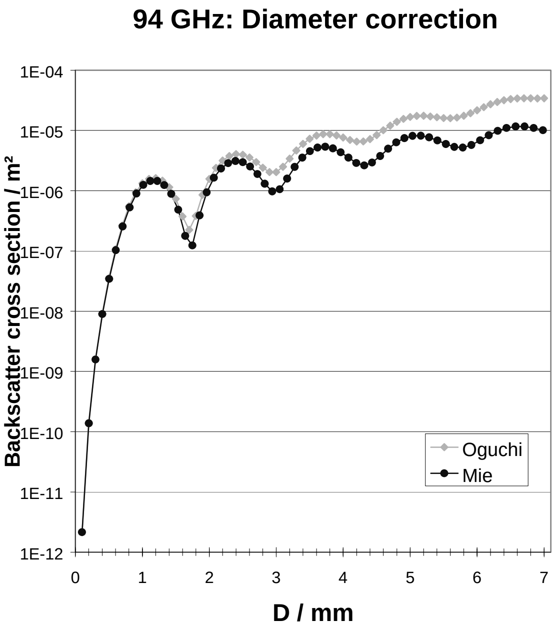


**Fig. 4.** Comparison of backscatter cross section versus drop diameter at 94 GHz calculated by Oguchi and Mie code, respectively. For the Mie calculation, the original diameter has been corrected by multiplying by $\beta^{1/6}$(see Eqs. (1) and (8)).

## 5 Conclusions

In Tab. 1, the drop (equivolumic) diameters, at which the absolute value of the relative error (see Eq. (7)) of the calculated backscatter cross section with respect to the Oguchi backscatter cross section exceeds 20% for the first time, are listed for the three investigated transmitting frequencies. It is obvious from Tab. 1 that cloud droplets (and even drizzle droplets) are in the Rayleigh region so retrieval algorithms based on the Rayleigh approximation can be applied.

**Table 1.** Drop (equivolumic) diameter in [mm] where the absolute value of the relative error of the back scatter cross section (Eq. (7)) excesses 20 %

| Approximation / f | 10 GHz | 24 GHz | 94 GHz |
|---|---|---|---|
| Rayleigh | 3.2 | 1.6 | 0.9 |
| Rayleigh, cor. f. flat. | 3.8 | 1.7 | 1.0 |
| Mie | 5.0 | 3.4 | 1.4 |

Applying the correction for drop flattening according to Klugmann and Richter (1992) to the Rayleigh backscatter cross sections provides a small advantage for the investigated transmitting frequencies.

The high relative errors of the Mie backscatter cross section with respect to the values calculated by the Oguchi code seem to prevent the utilization of Mie scatter code as an approximation of the more computing time consuming Oguchi code. But the fact that a significant contribution to this relative error is due to a shift in the location of the local minima and maxima of the backscatter cross section function gives some hope.

Applying a correction to the drop diameter used for Mie calculation provides a significantly improved match between the Mie and the Oguchi results. However, there is a difference in the backscatter cross section from both methods left. This difference increases with increasing drop size.

## 6 Outlook

The correction factor for drop ds applied to the Mie backscatter cross section has to be investigated for a higher number of temperature values and transmitting frequencies. Furthermore, the remaining deviation in the values calculated by Mie and Oguchi algorithm, respectively, has to be investigated. In addition to this, other relationships for the definition of the drop flattening (see Eq. (1)) might be introduced.

In a future step the algorithms will be applied to spectra from radar profilers operating at K-band and W-band.

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