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Estimation of Doppler Spectrum Parameters: Comparison between FFT-based Processing and Adaptive Filtering Processing

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1 Introduction

A new method to estimate the Doppler spectrum parameters of atmospheric signals based on adaptive filtering is presented. The new method is compared with the traditional FFT-based method. In order to do so both methods have been applied to process the same atmospheric data set obtained by the FM-CW polarimetric Doppler radar TARA (Transportable Atmospheric Radar) developed by IRCTR.

In section 2 the adaptive filtering processing is described. The details of the FFT-based algorithm will not be described. The reader is referred to Doviak and Zrnic (1993) for an extensive discussion. Section 3 describes the practical implementation of both algorithms in the radar. In section 4 the results of both algorithms are discussed and compared. Section 5 outlines the advantages and disadvantages of both methods.

2 Adaptive Filtering Processing

2.1 Signal and noise models

We assume that the backscattered signal from atmospheric objects placed at a particular range has the form, after discretization, of a complex autoregressive series of first order Z_t, with an unknown complex coefficient α:

$$\begin{aligned} Z_{t+1} &= \alpha Z_t + \varepsilon_{t+1} \\ \left\langle \varepsilon_i \varepsilon_j^* \right\rangle &= \sigma_\varepsilon^2 \delta_{ij} \end{aligned} \tag{1}$$

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Where t is discrete time, ε_t is a normal white random series with variance σ_ε^2, α is a complex parameter characterizing the signal, * denotes conjugation, $\langle\ \rangle$ means averaging over the number of samples and δ_{ij} is the Kronecker symbol.

The spectrum of signal (1) is symmetric relative to the average Doppler frequency and can be represented by the following formula, Yaglom (2004):

$$F_z(\omega) = \frac{\sigma_\varepsilon^2}{\left|1-\alpha e^{-i\omega}\right|^2} \tag{2}$$

$$\sigma_z^2 = \frac{\sigma_\varepsilon^2}{1-|\alpha|^2} \tag{3}$$

Where σ_z^2 is the power (variance) of the signal Z_t and ω is an angular frequency which lies within the range $[-\pi, \pi]$. A frequency equal to $-\pi$ or π of the discrete signal corresponds to the Nyquist frequency of the continuous signal. When $/\alpha/\rightarrow 1$, being less than 1, the signal can be approximated by a quasi-sinusoidal random series. As shown in Appendix A, the mean frequency of the spectrum, $\langle\omega\rangle$ coincides with the frequency of the maximum. The standard deviation of the spectrum width depends only on $/\alpha/$:

$$\begin{aligned} \langle\omega\rangle &= \Lambda = \arg(\alpha) \\ \Delta\omega &= \left\langle (\omega-\Lambda)^2 \right\rangle = \frac{\pi^2}{3} - 4\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2} |\alpha|^k \end{aligned} \tag{4}$$

In order to estimate the mean Doppler frequency and the Doppler spectrum width only the estimation of α is required.

We assume now that the atmospheric signal Z_t is measured mixed with uncorrelated additive normal complex white noise η_t, having variance σ_η^2. The measured signal X_t is therefore:

$$X_t = Z_t + \eta_t \quad (5)$$

Where:

$$\langle \eta_i \eta_j^* \rangle = \delta_\eta^2 \delta_{ij}; \langle \varepsilon_i \eta_j \rangle = \langle Z_i \eta_j \rangle = 0 \quad (6)$$

Using (1) and (5), the equation that relates measured and desired signals in two consecutive time instants can be written as:

$$X_{t+1} = \alpha Z_t + \varepsilon_{t+1} + \eta_{t+1} \quad (7)$$

2.2 The adaptive filter estimation

The adaptive filter estimator can be divided in two parts, the estimation part and the filtration part.

If we assume that both signals X_t and Z_t are known the estimation of the parameter α that is optimal in terms of maximum a-posteriori probability is given by the following recurrent equations, Liptzer and Shiryaev (1977):

$$\hat{\alpha}_{t+1} = \hat{\alpha}_t + \vartheta_t \frac{Z_t^* \left(X_{t+1} - \hat{\alpha}_t Z_t \right)}{\sigma_\varepsilon^2 + \sigma_\eta^2 + \vartheta_t |Z_t|^2}$$
$$\vartheta_{t+1} = \vartheta_t - \vartheta_t^2 \frac{|Z_t|^2}{\sigma_\varepsilon^2 + \sigma_\eta^2 + \vartheta_t |Z_t|^2} \quad (8)$$

Where ^ denotes the estimation. ϑ_t can be interpreted as the variance of the estimation of $\hat{\alpha}$.

If we assume now that α is known, the optimal recurrent filtration of the signal is given by the following differential equations, representing a particular case of the Kalman filter (Liptzer and Shiryaev, 1977):

$$\hat{Z}_{t+1} = \alpha \hat{Z}_t + \frac{\left(\sigma_\varepsilon^2 + |\alpha|^2 R_t \right)\left(X_{t+1} - \alpha \hat{Z}_t \right)}{\sigma_\varepsilon^2 + \sigma_\eta^2 + |\alpha|^2 R_t}$$
$$\vartheta_{t+1} = \left(\sigma_\varepsilon^2 + |\alpha|^2 R_t \right) - \frac{\left(\sigma_\varepsilon^2 + |\alpha|^2 R_t \right)^2}{\sigma_\varepsilon^2 + \sigma_\eta^2 + |\alpha|^2 R_t} \quad (9)$$

Here R_t is an average square of the filtration error. (9) actually represents a digital linear filter with one complex pole and a variable amplification coefficient.

The mutual substitution of (8) and (9) leads to a system of equations for the adaptive filter. The full system of equations that describes the adaptive estimator is given by the following formulas:

$$\hat{\alpha}_{t+1} = \hat{\alpha}_t + \gamma_{t+1} \hat{Z}_t^* \left(X_{t+1} - \hat{\alpha}_t \hat{Z}_t \right) \quad (10.a)$$

$$\gamma_{t+1} = \frac{\gamma_t}{1 + \gamma_t |Z_t|^2} \quad (10.b)$$

$$\hat{Z}_{t+1} = \hat{\alpha}_t \hat{Z}_t + P_{t+1} \left(X_{t+1} - \hat{\alpha} \hat{Z}_t \right) \quad (10.c)$$

$$P_{t+1} = \frac{\hat{Q}_t \left(1 - |\hat{\alpha}_t|^2 \right) + P_t |\hat{\alpha}_t|^2}{1 + \hat{Q}_t \left(1 - |\hat{\alpha}_t|^2 \right) + P_t |\hat{\alpha}_t|^2} \quad (10.d)$$

Here $\gamma_t = \frac{\vartheta_t}{\sigma_\varepsilon^2 + \sigma_\eta^2}$ and $P_t = \frac{R_t}{\sigma_\eta^2}$. In (10), we replace the signal to noise ratio $Q = \frac{\sigma_z^2}{\sigma_\eta^2} = \frac{\sigma_\varepsilon^2}{\sigma_\eta^2 \left(1 - |\alpha|^2 \right)}$ by its estimation $\hat{Q}_t$, which can be measured in advanced or obtained during ongoing measurements:

$$\hat{Q}_t = \frac{\sum_{i=1}^{t} |X_i|^2}{\sum_{i=1}^{t} |\eta_i|^2} - 1 \quad (11)$$

Where η_i are independently measured noise samples. In order to use (10) the following initial conditions can be applied: $\hat{a}_0 = 0; \hat{Z}_0 = 0; \gamma_0 = X_0^{-2}; P_0 = 1$.

(10) Provides the simultaneous optimal estimation of α (Eq. 10a, 10.b) and the signal (Eq. 10.c, 10.d). Therefore using (10) a recurrent estimation of α can be carried out while new data is acquired by the radar. After reasonable averaging, the mean Doppler frequency and spectrum width can be computed according to (4). It is shown in appendix B that if the signal is described by an autoregressive equation of first order, the estimation in (10) is asymptotically non-biased.

3 Implementation of the algorithms

The starting point for both algorithms is a data matrix which columns are range dependent. The rows are the discrete time series used in the processing. Clutter has been previously removed.

The first step in the FFT-based algorithm is to perform an FFT through the rows to obtain the Doppler spectrum and to calculate the power per Doppler cell. Doppler cells below a pre-established clipping level (Usually 10 dB above the estimated noise floor) are then suppressed in order to diminish the influence of noise spikes in the calculation of the Doppler parameters. The Doppler parameters are therefore calculated using only the Doppler cells with power above the clipping level. As for the adaptive algorithm the Doppler parameters are calculated directly from the time series using the equations developed in section 1. The reflectivity is calculated by subtracting the estimated noise power measured a priori from the measured signal power and applying a calibration factor.

4 Comparison of the two algorithms

This section discusses the results obtained by both algorithms. Due to the limited space and the poor resolution obtained in grey scale the actual processed data sets are not shown here. The reader can contact the correspondent author for samples.

The number of operations required to obtain the Doppler parameters using the FFT-method is roughly $6N*log_2(N)$, where N is the number of sweeps used. The number of operations required for the adaptive processing is instead just $41N$. Therefore the adaptive processing is much faster, especially when a large number of sweeps are used to compute the parameters. This gives a significant advantage when real-time processing is required.

Both algorithms assume that the atmospheric signal is stationary and symmetrical respect to the mean Doppler velocity. However atmospheric signals can be considered stationary just for a limited period of time since after that the signal becomes uncorrelated. In the case of TARA, an S-band radar, the signals are considered to remain stationary for a period of approximately half a second (512 samples with a sweep period of 1 ms). That limitation poses a serious drawback for the adaptive processing. In eq. (4) it is shown that the mean Doppler velocity depends just on the phase of the parameter α, while the Doppler spectrum width depends on the modulus of α. While the phase of α converges relatively fast to the asymptotic value and therefore a good estimation of the mean Doppler velocity is obtained this is not the case for the modulus. Moreover, the k-factor dependence of the estimation of the Doppler spectrum width propagates the small errors in the estimation of the Doppler velocity. Therefore in most of the cases an overestimation of the Doppler spectrum width, especially for wide spectra, has been observed. In Fig. 1 this can be observed.

The performance of the adaptive processor is clearly better than the FFT-based algorithm if clipping is not applied since the adaptive processor filters out most of the noise that contaminates the measurements. However, if clipping is applied the performance is similar since the clipping of the signal acts effectively as an adaptive filter. However, if the SNR is very low, i.e. all the Doppler cells are below the clipping level, the estimation of the Doppler parameters cannot be performed using the FFT-based method whereas the adaptive method is still able to estimate the mean Doppler velocity (See Table 1 and Fig. 2). Moreover due to the suppression of Doppler cells below the clipping level in the FFT-based algorithm, there is an underestimation of the signal power, and therefore of the reflectivity.

Table 1. Processing parameters. The SNR was set to 0.09

Parameter	Given	Estimated Adaptive processing	Estimated Doppler processing with 10 dB clipping
Signal Power	3.196638	3.284946	NaN
Mean Doppler Velocity	1	1.005346	NaN
Doppler Spectrum width	0.3	0.303753	NaN

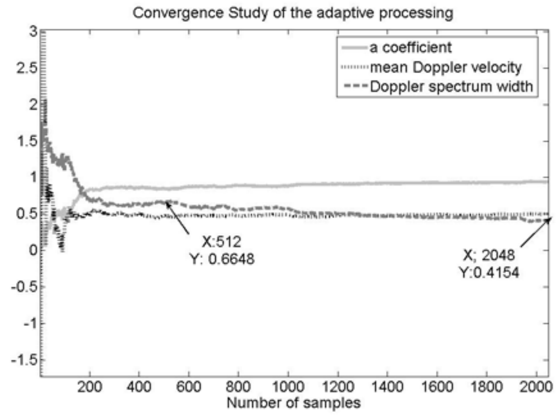

Fig. 1 Convergence study of a synthetic atmospheric signal: parameters: signal to noise ratio 0.5, mean frequency (normalized to Nyquist [-π,π]) 0.5, Doppler spectrum width (normalized) 0.4.

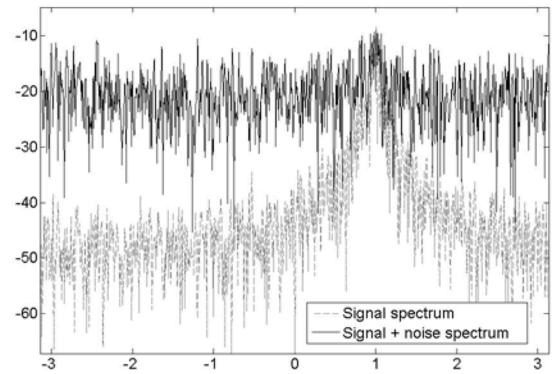

Fig 2 Synthetic atmospheric signal spectrum with noise and without. The clipping level used in the FFT-based processing was 10 dB above noise floor

5 Conclusion

A new method to calculate the Doppler spectrum parameters has been presented. The performance of the new method has been compared with the traditional FFT-based method. Two main advantages of the new method can be pointed out: It requires less operations and therefore it is faster, which is important in real time applications, and it is able to estimate accurately the mean Doppler velocity even in the case of very low signal to noise ratios where the traditional FFT method cannot carry out the estimation because the signal is below the clipping level. The major drawback of this method though, is the overestimation of the Doppler spectrum width due to slow velocity of convergence to the asymptotic level.

The signal model for the atmospheric signal in the adaptive processing is a simple complex autoregressive series of first order. It is expected that more complex models would lead to better performance.

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Appendix A Mean square of an autoregressive spectrum

The spectrum of a complex auto-regressive process $Z_{t+1} = \alpha Z_t + \varepsilon_{t+1}$ is represented as, Yaglom (2004):

$$F_z(\omega) = \frac{\sigma_\varepsilon^2}{\left|1-\alpha e^{-i\omega}\right|^2} = \frac{\sigma_\varepsilon^2}{1-|\alpha|^2 - 2|\alpha|\cos(\omega-\Lambda)} \quad \text{(A1)}$$

Where $\alpha = |\alpha| e^{i\Lambda}$. The mean spectrum width is denoted as a ratio between two integrals:

$$\Delta\omega = \left\langle (\omega-\Lambda)^2 \right\rangle = \frac{\int_{\Lambda-\pi}^{\Lambda+\pi} (\omega-\Lambda)^2 F_z(\omega) d\omega}{\int_{\Lambda-\pi}^{\Lambda+\pi} F_z(\omega) d\omega} = \frac{\int_{-\pi}^{\pi} \frac{\omega^2 d\omega}{1+|\alpha|^2 - 2|\alpha|\cos\omega}}{\int_{-\pi}^{\pi} \frac{d\omega}{1+|\alpha|^2 - 2|\alpha|\cos\omega}} \quad \text{(A2)}$$

To calculate the integrals in (A2) ω^2 can be expanded into a Fourier series within the range $-\pi, \pi$:

$$\omega^2 = \frac{\pi^2}{3} - 4\sum_{k=1}^{\infty} \frac{(-1)^{k+1}\cos(k\omega)}{k^2} \quad \text{(A3)}$$

The utilization of (A3) and the tabulated integral, Gradshteyn and Ryzhik (1980):

$$\int_{-\pi}^{\pi} \frac{\cos(k\omega) d\omega}{1+|\alpha|^2 - 2|\alpha|\cos\omega} = \frac{\pi}{1-|\alpha|^2}|\alpha|^k \quad \text{(A4)}$$

Leads to the equation for the spectrum mean square:

$$\Delta\omega = \left\langle (\omega-\Lambda)^2 \right\rangle = \frac{\pi^2}{3} - 4\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2}|\alpha|^k \quad \text{(A5)}$$

Appendix B Asymptotically non-biased estimation

The equations in (10) in the limit case, when t→∞, can be rewritten as:

$$\begin{aligned}
\frac{\alpha_\infty}{\gamma_{t+1}} &= \frac{\alpha_\infty}{\gamma_t} + \hat{Z}_t^* X_{t+1} \\
\frac{1}{\gamma_{t+1}} &= \frac{1}{\gamma_t} + \left|\hat{Z}_t\right|^2 \\
\hat{Z}_{t+1} &= (1-P_\infty)\alpha_\infty \hat{Z}_t + P_\infty X_{t+1} \\
P_\infty &= \frac{Q(1-|\alpha_\infty|^2) + P_\infty|\alpha_\infty|^2}{1+Q(1-|\alpha_\infty|^2) + P_\infty|\alpha_\infty|^2}
\end{aligned} \quad \text{(B1)}$$

Where symbol ∞ is used to define a constant value when *t*→∞. We will show that $\alpha_\infty = \alpha$ and therefore the Doppler parameters are asymptotically non-biased. Averaging over a great number of samples the first and the second equation from (B1), and substituting X_t for (7) when *t*→∞, leads to:

$$\alpha_\infty = \alpha \frac{\left\langle \hat{Z}_t^* Z_t \right\rangle_\infty}{\left\langle \left|\hat{Z}_t\right|^2 \right\rangle_\infty} \quad \text{(B2)}$$

To evaluate the correlation $\left\langle \hat{Z}_t^* Z_t \right\rangle_\infty$ we multiply the third equation in (B1) by the signal equation (7) and average over the samples. The stationary solution has the following form:

$$\left\langle \hat{Z}_t^* Z_t \right\rangle_\infty = \frac{P_\infty \sigma_z^2}{1-(1-P_\infty)\alpha\alpha_\infty^*} \quad \text{(B3)}$$

It follows from (B2) and (B3) that the ratio in the right part of (B2) is a real value and so $\arg\alpha_\infty = \arg\alpha$. That means that the estimation of the average frequency is non-biased.

In order to compute $\left\langle \left|\hat{Z}_t\right|^2 \right\rangle$, we multiply the third equation from (B1) by the conjugated one and average over the samples. For the stationary case solution it will be:

$$\left\langle \left|\hat{Z}_t\right|^2 \right\rangle_\infty = \frac{P_\infty(\sigma_z^2 + \sigma_\mu^2)}{1-(1-P_\infty)^2|\alpha_\infty|^2} + \frac{2P_\infty(1-P_\infty)|\alpha||\alpha_\infty|}{1-(1-P_\infty)^2|\alpha_\infty|^2}\left\langle \hat{Z}_t^* Z_r \right\rangle \quad \text{(B4)}$$

Using mutual substitutions (B2), (B3), (B4) and the last equation from (B1), it can be obtained $|\alpha_\infty| = |\alpha|$. Since the estimation of the spectrum width is determined by the absolute value of the complex number a, the estimation is asymptotically non-biased.