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Adaptive clutter identification and filtering using subspace processing technique

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1 Introduction

Clutter contamination is common in radar observations and is one of the major factors impacting quantitative applications. This can be particularly severe for ground based weather radar when observing the precipitation at low attitude and close range. Strong clutter can project observations with intense reflectivity as well as bias the Doppler moments. The NSF engineering research center of collaborative adaptive sensing of atmosphere (CASA) is developing a new sensing paradigm using short range radar to observe the lower troposphere (McLaughlin et al. 2005). Clutter will be an important aspect to deal with at close ranges.

To discriminate and mitigate clutter contamination is a long standing effort in radar community. Essentially the clutter contamination is discriminated from genuine weather signals by two assumed properties of clutter spectra: zero mean Doppler velocity and narrow spectrum width. Therefore high pass notch filter with a narrow stop band centered at zero frequency is commonly used. Its pass band should be steep enough to obtain both good clutter suppression and acceptable distortion on weather signal.

Nevertheless, the mean velocity of clutter is not always zero owing to the presence of intermittent clutter and the spectrum width can also change, e.g. owing to various scan rates (Doviak and Zrnic 1993). These susceptible variations require different design for the stop band of notch filters. However, clutter generally has narrower spectra than weather signal. Focusing on this property, a singular value decomposition (SVD) based subspace processing technique is developed in this paper for clutter identification and mitigation. It is shown that this approach is immune to the specific mean velocity and spectrum width of clutter. Simulation demonstrates that this new filter can perform fairly well at high clutter-to-signal ratio (CSR).

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2 Signal Entropy and Clutter Identification

Precipitation is composed of a large number of hydrometeors extending with different scattering amplitudes and moving at different velocities. The radar echo from precipitation volume can be characterized as a complex stochastic signal with Gaussian shaped distribution. Its autocorrelation can be expressed by (Bringi and Chandrasekar 2001)

$$R(n) = S_0 \exp\left(-\frac{8\pi^2 \sigma_v^2 n^2 T_s^2}{\lambda^2} \right) \exp\left(-\frac{j4\pi v_0 n T_s}{\lambda} \right) \quad (1)$$

and the corresponding power spectral density follows

$$S(v) = \frac{S_0}{\sqrt{2\pi}\,\sigma_v} \exp\left(-\frac{(v - v_0)^2}{2\sigma_v^2} \right) \quad (2)$$

In the above equations, T_s is the pulse repetition time (PRT), λ is the carrier wavelength, S_0 is the signal power, v_0 is the mean Doppler velocity, which can be estimated by

$$v_0 = -\frac{\lambda}{4\pi T_s} \arg[R(1)] \quad (3)$$

and σ_v is the spectrum width, which can be estimated by

$$\sigma_v = \frac{\lambda}{2\sqrt{2}\pi T_s} \sqrt{\ln\left|\frac{R(0)}{R(1)}\right|} \quad (4)$$

Equations (1) and (2) can also be used to characterize clutter. For weather signal, typically its mean velocity is nonzero due to the terminal velocities of falling hydrometeors. The hydrometeor particles distributed in the resolution volume contribute to a wide Doppler spectrum width (>1m/s). In contrast, clutter signal is mostly attributed to hard targets, such as buildings, trees, and maybe moving vehicles. Its spectrum width will be smaller than that of weather signal (<1m/s).

Narrow spectrum width means a signal having long correlation time, or high regularity (in contrast to white noise). Applying singular value decomposition (SVD) to the sample matrix **A** of a length-*N* signal, we have

$$\mathbf{A}_{M\times N} = \mathbf{U}_{M\times M}\mathbf{\Lambda}_{M\times N}\mathbf{V}^{\mathbf{H}}_{N\times N} \quad (5)$$

where M and N express the size of the sample matrix ($M<N$), the superscript $^{\mathbf{H}}$ stands for conjugate transpose, and Λ is a diagonal matrix of singular values $\{\lambda_i\}_{i=1...M}$. For signals of high regularity, only a few λ_i can have significant values. For example, a sinusoid of single frequency has a rank of 1 while white noise has full rank. For stochastic signals as specified by (1) and (2), that of wider spectrum width is expected to have more significant singular values compared to that of narrower spectrum width. However, the rank of sample matrix **A** is not sufficiently precise to describe the stochastic signals. Instead, the entropy of **A** is defined as,

$$I = -\sum_{i=1}^{M}\lambda_i^{'}\log_2\lambda_i^{'} \quad (6)$$

where it is assumed that $M<N$, and $\lambda_i^{'}$ is the i-th normalized singular value, i.e.,

$$\lambda_i^{'} = \lambda_i^2 / \sum_{k=1}^{M}\lambda_i^2 \quad (7)$$

Fig.1 shows the singular values for signals having Gaussian spectra and the associated entropy. Note that both signal power and mean velocity are not related here. The spectrum widths of 0.25m/s and 4m/s are typical values for clutter and weather signal, respectively. The signal entropy increases monotonously with its spectrum width. The entropy can resolve the change of spectrum width very well within the range σ_v<2m/s. Although Gaussian spectrum is assumed in generating Fig.1 for simulation, it does not require the signal to have Gaussian shaped spectrum because the entropy defined in (6) just reflects the signal rank in a precise manner.

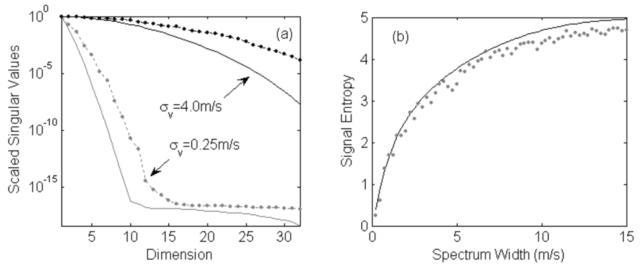


Fig.1. Comparison of singular value distributions and entropies: (a) The singular value distribution (scaled with respect to the maximum singular value) for signals with σ_v of 0.25m/s and 4m/s; (b) The signal entropy. The solid lines correspond to theoretical curves, while the dotted lines are based on simulation (N=128 and M=32).

It can be expected that the most significant singular values are contributed by clutter if clutter dominates the radar echoes; accordingly the signal entropy will be small. Therefore the clutter contamination can be identified through the signal entropy. In this paper, we use the spectrum width of 1m/s as the boundary between clutter and weather signal. The threshold on signal entropy is chosen at 1.4. Note that this identification works on single polarization observations and allows variation in the mean velocity of clutter. It can be used to replace the clutter map for a specific radar. Even when dual polarization parameters are available, the use of entropy threshold overcomes the complicated aggregation algorithm on multiple radar parameters.

3 Subspace Processing and Adaptive SVD Filter

The singular values are related to the magnitudes of a signal on the corresponding canonical dimensions. At small entropy, the signal power is concentrated on a few dimensions, as indicated by the singular values that are significant. If its rank is known, a signal can be efficiently represented using these singular values on the corresponding subspace. Therefore, in the presence of strong clutter, the clutter can be approximated on a subspace with a small number of dimensions, i.e., with reduced rank. On the other hand, weather signal and noise span over a subspace with more dimensions and can be approximated on the complementary subspace.

Essentially all the linear filters can be interpreted by subspace processing techniques. A general form of linear filtering on discrete time signal **x** can be written as

$$\mathbf{y} = \mathbf{T}\mathbf{x} \quad (8)$$

where the filter **T** represents a transform on **x** from the measurement space to another vector space. Regarding radar signal for Doppler processing, **x** is composed of N complex samples at interval of PRT. The filter **T** is expected to be an orthogonal projection that maps a signal of multiple components to some other vector space where the components of interest concentrate in a selective subspace and hence can be further separated.

Notch filter removes signal components within a narrow frequency band around zero in the frequency space. It is equivalent to a projection of **x** onto a subspace spanned by sinusoids of high frequencies. The notch filter assumes that the basis is known a priori. If the spectrum width, the mean velocity, or the power of clutter changes, the notch filter needs to be redesigned accordingly. In contrast to this, eigen filter is able to adaptively choose the working bases, which corresponds to the eigenvectors of its sample correlation matrix. Such filter has been used to filter clutter for high frequency color flow mapping system (Kruse and Ferrara, 2002).

3.1 Adaptive Truncated SVD Filter

The subspace processing can be generally and rigorously described by the Eckart-Young Theorem (Tufts at el., 1982). Given a sample matrix as shown in (5), the Eckart-Yong Theorem states that, for an arbitrary matrix **B** of rank m,

$$\|\mathbf{A}-\mathbf{A}_m\| \le \|\mathbf{A}-\mathbf{B}\| \quad (9)$$

where

$$\mathbf{A}_m = \mathbf{U}\mathbf{\Lambda}_m\mathbf{V}^{\mathbf{H}} \quad (10)$$

and $\mathbf{\Lambda}_m$ is its singular value matrix truncated at m-th largest singular value. Therefore, $\mathbf{A}_m$ is the beast least squares approximation of lower rank m to the sample matrix **A**. If m is known for clutter, the clutter components can be estimated using (10) and the weather signal along with noise can be further approximately separated using the complementary part. This general subspace processing technique is termed as truncated singular value decomposition (TSVD).

To best approximate the clutter contamination, its dimension m is to be known. The dimension m largely depends on the spectrum width of clutter, while not on its mean velocity. Therefore the success of TSVD filter comes down to the adaptive determination of m. The signal entropy as defined in (6) is a good candidate parameter to this end. As long as the signal entropy is smaller than a threshold, which is chosen at 1.4 as explained in the previous section, it indicates that clutter dominates the radar echoes. Then the largest singular value is taken out, and the signal entropy is to be checked again using the singular values left. Provided that the spectrum width of clutter is indeed less than 1 m/s, this selection of dimension m leads to a TSVD filter that is able to adaptively adjust itself tuning to the true mean velocity and spectrum width of the clutter.

Implementation of this adaptive TSVD filter requires a sample matrix. To construct **A** from a single observation vector **x**, we can arrange **x** into a Hankel matrix up to lag M (Tufts at el., 1982; Hansen and Jensen, 1998). The matrix has identical values along its anti-diagonal. Subject to the adaptive TSVD processing, the best approximation $\mathbf{A}_m$ is arrived at for clutter. The clutter vector is obtained by averaging over the anti-diagonals of $\mathbf{A}_m$ and the filtered version of **x** is available by subtracting clutter from the original signal. Dologlou and Carayannis (1991) showed that such averaging is equivalent to subtracting from the original signal the information contained in the residual eigenvalues. The TSVD processing can be interpreted as a parallel array of analysis-synthesis FIR filter pairs (Hansen and Jensen, 1998).

3.2 Simulation

Simulation is used to evaluate the clutter filter using the adaptive TSVD processing. The weather signal is simulated with a Guassian spectrum of 5m/s mean and 4m/s width and with 10dB SNR relative to a fixed noise floor. The estimates of reflectivity (Z), the mean velocity (V) and the spectrum width (σ_v) are computed from filtered signals using the pulse-pair method (3-4). At CSRs from 10dB up to 80dB, the mean and standard deviation of these estimation biases are plotted in Fig.2. The threshold 1.4 is used on the signal entropy to determine the clutter dimensions. During this process, because the largest singular value is removed, the loss in the number of residual singular values is compensated in the calculation of signal entropy. Also the singular values in transient region, which are greater than the threshold, are regarded as clutter components.

Fig.2 shows that the adaptive TSVD filter results in power loss progressively with CSR. The physical interpretation is that there exists overlap between clutter subspace and signal subspace. As CSR increases, more singular values are removed and the filter suffers larger power loss. Due to the same reason, the estimated mean velocity suffers apparent biases, likely because the residual spectrum is not symmetric anymore. The biases due to overlap of signal spectrum and clutter spectrum also exist in notch filters. Using the TSVD filter, the biases are fairly small if CSR is less than 40dB. At large CSR, however, the filter generally still works in estimating the weather signal power.

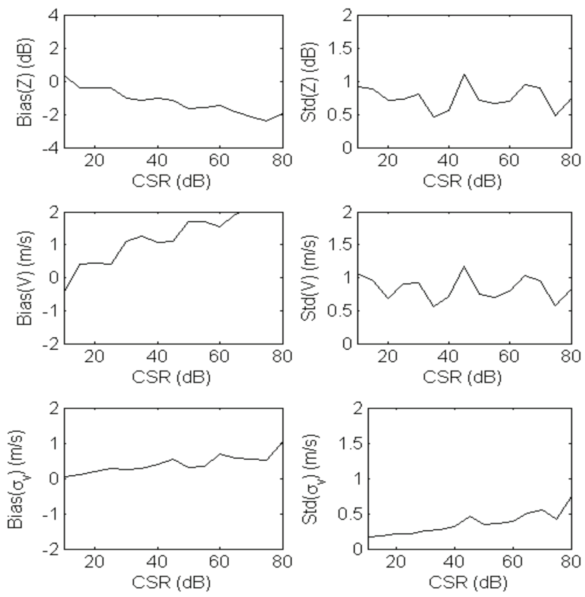


Fig.2. Clutter mitigation performance of the adaptive TSVD based on 100 realizations. Clutter is simulated with zero mean velocity and σ_v of 0.25m/s; weather signal is simulated with mean velocity of 5m/s and σ_v of 4m/s; SNR=10dB, N=128, and M=32.

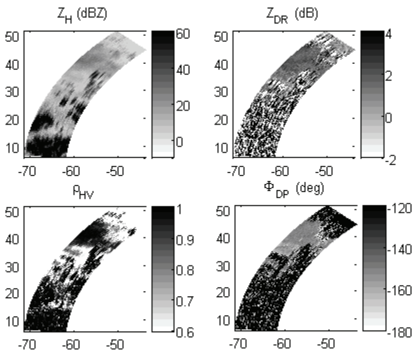
3.3 Case Study

Radar observations by the CSU-CHILL radar are used to preliminarily test the adaptive TSVD filter. The original radar parameters and these after filtering are shown in Fig.3. Only the observations that are identified as clutter using the entropy threshold go through the TSVD filter. Essentially all the susceptible clutter returns are removed. The residual reflectivity is attributed to noise, as evident in the extreme low co-polar correlations. It should be noted that the test is not conclusive, because the clutter itself is isolated from weather signals. Nonetheless, it appears that the clutter is correctly identified.

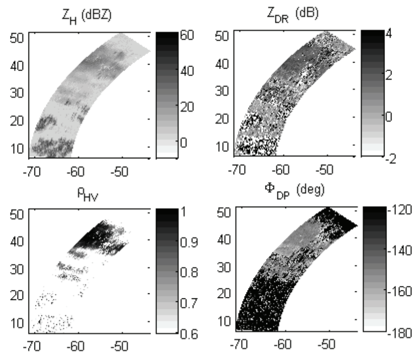
3.4 Limitations

The biases shown in Fig.2 imply that the signal subspace partly overlaps with the clutter subspace. The TSVD filter does not attempt to compensate the removal of such overlap. Fig.4 shows the reflectivity biases at fixed CSRs with respect to the mean velocity and spectrum width of weather signal. The biases are severe when the mean velocity of weather signal is low. Obviously all the nonparametric filtering approaches suffer this limitation. Model based parametric method, such as Gaussian Model Adaptive Processing (GMAP), compensate the notched components using fitted weather spectrum (Saggia and Pasarelli, 2004). However, it strongly relies on the validity of the Gaussian model. The window effect also limits the use of GMAP at higher CSR.

The adaptive TSVD filter assumes good separation in spectra of clutter and signal. When the weather signal has a spectrum width less than 2m/s, larger biases are expected. This limitation is attributed to the determination of clutter dimensions, which is likely inaccurate in this situation.



(a)



(b)

Fig.3. PPI plot of dual polarization radar parameters estimated (a) before and (b) after the adaptive TSVD filter. The observations were collected by the CSU-CHILL radar at simultaneous transmission mode on 2145 UTC 24 June 2004. The elevation angle was set at 0.49°, PRF=1000Hz, N=128, and M=32.

4 Summary

An adaptive subspace processing technique is introduced and explored in this paper. Signal entropy that is defined over the singular values of sample matrix of observations is used to identify clutter contamination and further used to adaptively determine the clutter dimensions. Truncated SVD is then applied to approximate and remove clutter. Similar to eigen filter, the TSVD filter chooses bases adaptively based on received signal. In this sense it is an extension to the notch filters and as a result, the filter can deal with higher CSR. In the presence of velocity overlap, TSVD suffers power loss of weather signal, nonetheless it is the optimal nonparametric filtering approach as indicated in (9) given that the clutter dimensions are known. The adaptive determination of clutter dimensions is not optimal; however, if the spectrum width is sufficiently large for the weather signal, the adaptive TSVD filter can be potentially used to mitigate strong mainlobe clutter and intermittent clutter.

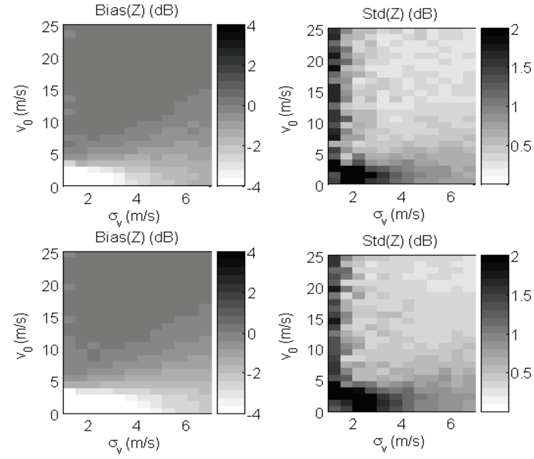


Fig.4. Clutter mitigation performance of the adaptive TSVD filter at 40dB CSR (the top row) and 60 dB CSR (the bottom row) based on 100 realizations. The spectrum width and mean velocity of weather signal are given in X- and Y-axis respectively. Clutter has zero mean velocity and σ_v of 0.25m/s. SNR=10dB, N=128, and M=32.

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