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Spectral processing of staggered PRT sequences to remove clutter and obtain polarimetric variables

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1 Introduction

The staggered pulse repetition time (PRT) technique (Sirmans et al. 1976, Zrnic and Mahapatra 1985) for the resolution of the range-velocity ambiguities in weather radars has reached a mature stage ready for operational application. The main difficulty with the staggered PRT method has been the clutter filtering. Sachidananda and Zrnic (2000) have proposed a spectral domain procedure which allows effective filtering of the ground clutter under certain conditions of the "narrow" signal spectra. This condition can be easily met in practice with proper choice of PRTs T_1 and T_2. Overall the best compromise between clutter filtering and extending the unambiguous range and velocity is for the stagger ratio, $\kappa = T_1/T_2 = 2/3$. At other stagger ratios the portion of the spectrum where signal can be recovered is smaller. In this paper we demonstrate how the complex Doppler spectrum of staggered PRT sequence can be obtained over 40% of the extended unambiguous velocity interval. Importance of spectral processing is increasing because it offers improvement of data quality, detection of tornadoes, and separation of some scatterer types; further, this capability just became available on the US National network of weather radars. Moreover, for filtering ground clutter out of the staggered PRT sequence spectral processing is needed. An added advantage of such processing is that in radars that simultaneously transmit horizontally and vertically polarized waves it is more efficient and accurate to estimate polarimetric variables from the complex spectra.

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2 The staggered PRT

In the staggered PRT technique (Zrnic and Mahapatra 1985) alternate pairs of echo samples are used to compute the autocorrelation estimates, R_1 at lag T_1 and R_2 at lag T_2 ($T_2 > T_1$). The difference in PRTs, (T_2-T_1), determines the extended unambiguous velocity, v_a, and is given by

$$v_a = \lambda / [4(T_2 - T_1)] \; ; \; T_1 < T_2 . \qquad (1)$$

Very good estimates of mean velocities are obtained if R_1 is used for computing an aliased velocity v_1 and the velocity from R_2 to de-alias v_1 over the unambiguous interval $\pm v_a$ (Sachidananda et al. 2001, Torres et al. 2004).

Concerning notation herein the lower case letters represent time domain quantities and the upper case letters spectral domain quantities. Vectors (column matrices) and matrices are represented by bold face letters. Filtering the ground clutter involves converting the staggered PRT echo sample sequence into a uniform PRT sequence by inserting zeros in place of missing samples (Sachidananda and Zrnic 2000); this uniform sequence we call the derived time series. To make this conversion T_1 and T_2 must be integer multiples of some basic PRT, T_u, so that $T_1 = n_1 T_u$, and $T_2 = n_2 T_u$, where n_1 and n_2 are integers. The stagger ratio is defined as, $\kappa = T_1/T_2 = n_1/n_2$. The spectrum of the derived time series, e, is a convolution of the signal spectrum with the spectrum of the code sequence, c_N (for example, $c_N(n)$ = [1010010100... etc.] for κ=2/3). The sequence length is $N = (n_1+n_2)L$, and L is the number of segments of the basic periodic part of the code c={10100}, which we will call the code kernel.

3 Spectral analysis

Assume that a uniform PRT sequence $s(nT_u)$ is observed at time intervals given by the code so that

$$e(nT_u) = c_N(n) \, s(nT_u). \qquad (2)$$

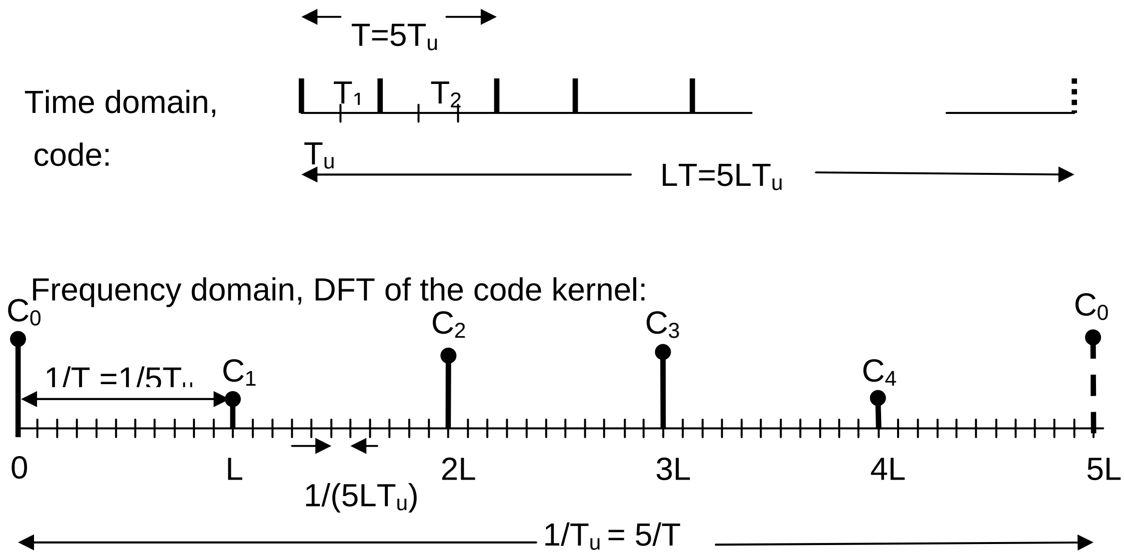

Fig .1 Relations between parameters of the code and its discrete Fourier transform.

Then the spectrum (DFT) of the derived (staggered) sequence is the convolution of the spectrum of the code with the spectrum of the uniform sequence.

The spectrum of the code $c_N(n)$ is comprised of the spectrum of the kernel c which has five coefficients uniformly spaced over the Nyquist interval ($1/T_u$). Between the uniformly spaced coefficients there are L-1 zero coefficients. This is illustrated in Fig. 1 where L=11. The figure helps understanding how various sinusoids contribute to the spectrum of the derived sequence. Any sinusoid at a frequency $0 \leq\ l \leq 5L$-1 of the spectrum of $s(nT_u)$ contributes to five replicas, one at its frequency (coefficient l) and the other four spaced L coefficients apart. The phases and amplitudes of these replicas are exactly the same as the replicas of the 0 frequency coefficient (only amplitudes are drawn in Fig.1). Thus any single line in the convolved spectrum could be comprised of additive contribution from up to five equally spaced sinusoids. Separation of these sinusoids is addressed next.

Let the DFT(c) of the kernel be $\mathbf{C} = [C_0, C_1, C_2, C_3, C_4]$; further the vector $\mathbf{C}$ is normalized so that its magnitude is one ($\sum|C_i|^2 = 1$). Then the convolution operation (which produces five replicas of the spectrum of sinusoids) is

$$\begin{bmatrix} E(k) \\ E(k+L) \\ E(k+2L) \\ E(k+3L) \\ E(k+4L) \end{bmatrix} = \begin{bmatrix} C_0 & C_4 & C_3 & C_2 & C_1 \\ C_1 & C_0 & C_4 & C_3 & C_2 \\ C_2 & C_1 & C_0 & C_4 & C_3 \\ C_3 & C_2 & C_1 & C_0 & C_4 \\ C_4 & C_3 & C_2 & C_1 & C_0 \end{bmatrix} . \begin{bmatrix} S(k) \\ S(k+L) \\ S(k+2L) \\ S(k+3L) \\ S(k+4L) \end{bmatrix}$$

; $0 \leq k \leq L$-1. (3)

This equation is compacted into the following matrix form

$$\mathbf{E}(k) = \mathbf{CS}(k) . \qquad (4)$$

In (4) $\mathbf{S}(k) = [S(k), S(k+L), S(k+2L), S(k+3L), S(k+4L)]^T$ represents the spectral coefficients at corresponding frequencies (superscript T signifies transpose and it is understood that the frequency number k is between 0 and L-1). Note that only sinusoids at five uniformly spaced frequencies convolve and thus contribute at these same frequencies. If a single sinusoid is present there would be one element in the S vector at the right side of (3), the other elements are 0, and the observed spectrum of the derived (staggered) sequence (the column E on the left) would represent the five replicas of that sinusoid.

The spectral coefficients of the code kernel are (5)

$\mathbf{C}=[C_0, C_1, C_2, C_3, C_4]=[2, 1+e^{-j2\alpha}, 1+e^{-j\alpha}, 1+e^{j\alpha}, 1+e^{j2\alpha}]/\sqrt{10}$, where $\alpha = 2\pi/5$, and division by $\sqrt{10}$ normalizes the vector $\mathbf{C}$. Of all code kernels (that allow both extension of unambiguous range and unambiguous velocity) the 10100 has the largest phase difference (72^o) between its spectrum coefficients. This large phase difference is the principal feature that allows separation of two overlaid spectral coefficients.

It can be verified, by inserting (5) into (3) that the rank of the convolution matrix is 2. This is to be expected as only two independent time samples (the two ones in c) are included in the computation of the DFT.

Although the convolution seems to hopelessly scramble the spectral coefficients, examination of (3) reveals that perfect deconvolution is possible if no more than two spectral coefficients are scrambled. That is the vector $\mathbf{S}$ (column in eq. 3) contains only two non-zero elements. This is equivalent to reducing $\mathbf{C}$ to a 2x2 matrix (by deleting any three rows and the corresponding columns) which is non-singular and hence the system of equation is solvable exactly. Such condition is often satisfied as explained next. Consider the 2:3 staggered ratio, 10 cm wavelength, and T_1 = 1 ms (unambiguous velocity v_{a1} =

25 m s^{-1}, unambiguous range r_{a1} = 150 km), T_2 = 1.5 ms (v_{a2} = 50/3 m s^{-1}, r_{a2} =225 km); then $v_a = v_{a1}v_{a2}/(v_{a1}-v_{a2})$ = 100 ms^{-1}. One fifth of v_a corresponds to the spacing $1/(T_1+T_2)$, (i.e., L coefficients out of 5L). If the weather spectrum were to span more than 2L coefficients (40 m s^{-1}) there would be triple overlap of some coefficients and these could not be perfectly retrieved. Otherwise Doppler spectrum occupying ≤ 40 m s^{-1} interval centered on the mean velocity can be perfectly retrieved. This is certainly a very liberal allowance considering that the largest median values of spectrum width are smaller than 6 m s^{-1} (Feng et al. 2004)

If the weather spectrum extends exactly over 2L coefficients (i.e., 40% of the interval 5L) then two weather spectral coefficients spaced L units apart will be combined in the convolution process i.e., vector **S** (has two non-zero elements). Hence the five linear equations represented by (3) are overdetermined. Exact inversion is possible if one knows where the original 2L contiguous spectral coefficients are located within the 5L coefficients. That is, one must know which two of the five elements of **S** in (3) to retain. An independent location can be obtained using magnitude deconvolution to determine the mean Doppler velocity from such deconvolved spectrum (Sachidananda and Zrnic 2000). That is, the magnitude of the $\mathbf{S_d}(k)$, $\{\mathbf{S_d}(k)^T = [S_d(k), S_d(k+L), S_d(k+2L), S_d(k+3L), S_d(k+4L)]$, is computed as

$$\text{abs}[\mathbf{S_d}(k)] = \text{abs}\ \{[\text{abs}(\mathbf{C})]^{-1}\text{abs}[\mathbf{E}(k)]\}, \qquad (6)$$

where the subscript d signifies that the spectrum coefficient comes from deconvolution. After the operation (6) is completed L times (once for each k) there would be L sets of five replicas, each set separated by one coefficient from its adjacent neighbor (spectrum line). Thus the recombined sequence of spectrum coefficients is $S_d(0)$, $S_d(1)$, $S_d(2)$... $S_d(L)$, $S_d(L+1)$, $S_d(L+2)$,... $S_d(2L)$, $S_d(2L+1)$, $S_d(2L+2)$........ $S_d(4L)$, $S_d(4L+1)$,... $S_d(5L-1)$.

If there are at least two spectral components (sinusoids) spaced a multiple of L coefficients apart, for example $S(k)$ and $S(k+L)$, the $S_d(k)$ will differ from $S(k)$ of the uniform PRT sequence and so would $S_d(k+L)$ from $S(k+L)$.

The mean velocity (frequency) of the deconvolved spectrum locates the center of the original spectrum. Suppose that this mean corresponds to the coefficient m and $m<L$, then the $\mathbf{S}(k)^T = [S(k), S(k+L), 0, 0, 0]$ should be used in (3) for $m \le k \le m+L/2$, (if L is odd subtract one from L and adjust so that all L coefficients are considered). For coefficients between m-L/2 and m the reconstruction should take $\mathbf{S}(k)^T = [S(k), 0, 0, 0, S(k+4L)]$.

4 Practical aspects

The reconstruction thus far considers a spectrum in which two coefficients are contiguous, i.e., separated by L lines. In some situations significant coefficients can be separated by 2L or 3L segments (larger circular separation is not possible). Such separation is often between coefficient of weather signal and ground clutter (or fast moving objects).

In cases that the ground clutter in the derived sequence overlaps weather spectra, clutter replicas must be removed (set to zero) to correctly locate the position of the weather spectrum via (6). Then the clutter complex spectral component and one weather component can be obtained. Thus the two strongest components can be retrieved provided that other components are negligible.

Al low SNRs reconstruction of spectra is difficult and more so is the separation of overlapping spectral components. Five noise components overlay each other and if these have powers comparable to the signal power the retrieval fails. Spectral components of comparable power but separated by more than 2L coefficients (e.g., one due to aircraft traffic the other due to weather) are also hard to separate because there might not be a simple way to determine the correct (original) location of these components.

Complex spectra are needed for computing the polarimetric variables after removal of ground clutter from the staggered PRT sequence. Further such spectra can be useful for detecting small tornadoes within the radar resolution volume.

Results (to be presented at this conference) indicate that the clutter filtering coupled with the spectrum recovery algorithm is very effective in processing staggered PRT sequence from dual-polarized radar. The fields of the polarimetric variables thus obtained at 0.44^o elevation exhibit spatial continuity and an order of magnitude reduction of clutter contaminated area.

8. References

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