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Comparison of two *X*-band DSD retrieval algorithms

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1 Introduction

One of the factors that limit the accuracy in estimating microphysical rain parameters needed in the validation of cloud model simulations and satellite rain retrievals of stratiform and convective precipitation is the lack of detailed knowledge of Drop Size Distribution (DSD).

Extensive research based on measured DSD spectra, suggests that, for short time periods proportionate with radar measurements, DSDs are more typically represented by a gamma distribution (Ulbrich 1983),

$$N(D) = N_0 D^{\mu} \exp(-\Lambda D) \quad (1)$$

where N_0 (mm$^{-\mu-1}$ m^{-3}) is the concentration number parameter, μ is the distribution shape parameter, Λ (mm^{-1}) is the slope term, and D (mm) is the equivalent volume drop diameter. Since the gamma DSD is described by three parameters, it requires three independent measurements, or relationships, to uniquely evaluate the parameter values.

In this study we investigate two retrieval techniques for estimating all three DSD parameters from X-band dual-polarization observations. The first retrieval technique is an adaptation for *X*-band of the technique proposed by Zhang et al. (2001), which is based on the two power-related radar parameters, the reflectivity at horizontal polarization (Z_H) and differential reflectivity (Z_{DR}). The technique uses a constrained μ–Λ relationship to derive the third DSD parameter.

The second method was initially implemented for *S*-band frequency by Bringi et al. (2002) to retrieve the three DSD parameter values on the basis of three polarimetric radar parameters: reflectivity (Z_H), differential reflectivity (Z_{DR}), and the specific differential phase (K_{DP}). In the following we describe the two techniques and evaluate their performance on the basis of synergistic X-band polarimetric observations with an S-band polarimetric radar and in situ disdrometer.

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2 Background

2.1 Polarimetric radar parameters

The polarimetric radar parameters that are most important for quantitative rain estimation are the horizontal polarization reflectivity, Z_H (mm^{6}m^{-3}), vertical polarization reflectivity (Z_V, mm^{6}m^{-3}) and differential reflectivity, Z_{DR} (ration of Z_H to Z_V in dB), and the specific differential phase shift, K_{DP} (° km^{-1}). These variables depend on the raindrop size distribution, DSD, and the drop scattering amplitudes as follows:

$$Z_{H,V} = \frac{4\lambda^4}{\pi^4 |K_w|^2} \int_{D_{\min}}^{D_{\max}} \left| f_{HH,VV}(D) \right|^2 N(D) dD \quad (2)$$

$$Z_{DR} = 10 \log\left(\frac{Z_H}{Z_V} \right) \quad (3)$$

$$K_{DP} = \frac{180\lambda}{\pi} \int_{D_{\min}}^{D_{\max}} \mathrm{Re}\left[f_{HH}(0,D) - f_{VV}(0,D) \right] N(D) dD \quad (4)$$

where D_{min} and D_{max} are the diameters of the smallest and largest drops in the distribution. The $f_{HH,VV}(D)$ and $f_{HH,VV}(0,D)$ are the backscattering and the forward scattering amplitudes of a drop at horizontal and vertical polarization, K_w is the dielectric factor of water, λ (cm) is the radar wavelength, and $N(D)$ (mm^{-1} m^{-3}) is the count of raindrop of size D. The $f_{HH,VV}(D)$ and $f_{HH,VV}(0,D)$ parameters depend on the assumed raindrop shape-size relationship as discussed in the subsequent section.

The horizontal polarization reflectivity, Z_{mH} (mm^{6}m^{-3}), and differential reflectivity, Z_{mDR} (dB), measured by the radar at range gate "r", are related to the corresponding equivalent (non-attenuated) radar parameters (Z_H and Z_{DR}) as follows:

$$Z_{mH}(r) = Z_H(r) \times 10^{-0.2\int_0^r A_H(s) ds} \quad (5)$$

$$Z_{mDR}(r) = Z_{DR}(r) \times 10^{-0.2\int_0^r A_{DP}(s)ds} \quad (6)$$

A_H and A_{DP} (dB km^{-1}) are the specific and differential rain-path attenuation, respectively.

2.2 Raindrop size distribution model

The raindrop size distribution model used in this research is the ''normalized gamma distribution'' function as presented in recent polarimetric radar rainfall studies (e.g., Testud et al. 2000):

$$N(D) = N_W f(\mu)\left(\frac{D}{D_0}\right)^{\mu} e^{-(4+\mu)\left(\frac{D}{D_0}\right)} \quad \left(mm^{-1}m^{-3}\right) \quad (7)$$

with

$$f(\mu) = \frac{6}{4^4}\frac{(4+\mu)^{(\mu+4)}}{\Gamma(\mu+4)} \quad (8)$$

where N_w (in mm^{-1}m^{-3}) is called the "normalized intercept parameter" and is the N_0 of an equivalent exponential DSD that has the same liquid water content (in gr m^{-3}) and raindrop volume diameter D_0 (in mm) as the gamma DSD.

The governing parameters of Gamma DSD model (N_W, D_0, and μ) are estimated from raindrop spectra in the following way. First we calculate the median mass diameter (D_m, in mm) and the water content (LWC, in gr m^{-3}). Subsequently, D_0 (in mm) is obtained on the basis of the following equation: $D_m(3.67+\mu)/(4+\mu)$, while N_w (mm^{-1}m^{-3}) is determined from liquid water content and D_0 as following:

$$N_W = \frac{3.67^4}{\pi\rho_w}\left[\frac{LWC}{D_0^4}\right]. \quad (9)$$

The shape parameter μ is then determined by minimizing (with respect to μ) the least square difference of calculated (from Eq. 10) versus sampled (from 3-minute averaged spectra) counts over a range of 20 drop diameter bins.

2.3 Simulation of radar parameters from DSD spectra

As shown by the integral equations [2-4], information on the DSD, as well as hydrometeors' shape-size relationship are needed to relate polarimetric radar measurements to precipitation and other radar parameters. In this study, we will use two raindrop shape-size relationships. The first relationship is the one given by Brandes et al. (2002):

$$r = 0.9951 + 0.0251D - 0.03644D^2 + 0.00503D^3 - 0.00025D^4 \quad (10)$$

The second is a linear relationship between r and D (D is in mm):

$$r = 1.03 - \beta D. \quad (11)$$

A point to note about this relationship is that β is variable and that this variability can be determined on the basis of polarimetric radar parameters.

3 DSD parameter retrieval

The difference between the scattering amplitudes at the two polarizations depends on the raindrop shape. Two methods originally developed for *S*-band dual polarization measurements are parameterized and evaluated here for the *X*-band frequency. The DSD retrievals named *constrain*-algorithm and β-method are reviewed in the sections that follow.

3.1 The *Constrain*-algorithm

The method starts with estimation of D_0 and LWC parameters based on relationships derived from scattering calculations using raindrop spectra and the Brandes et al. (2002) axial ratio model. We relate the non-attenuated *X*-band radar parameters (Z_H in mm^{6}m^{-3} and Z_{DR} in linear value) to the median-drop diameter (D_0 in mm) and liquid water content (LWC in gr m^{-3}) through the following best-fit relations:

$$D_0 = a + bZ_{DR} + cZ_{DR}^2 + dZ_{DR}^3. \quad (12)$$

$$LWC = AZ_H . 10^{\left(BZ_{DR} + CZ_{DR}^2 + DZ_{DR}^3 + EZ_{DR}^4\right)}. \quad (13)$$

The last parameter to be estimated is "μ". Analysis of DSD data revealed a good correlation between μ and Λ that led to the derivation of an empirical μ-Λ relation (Zhang et al. 2001).

$$\Lambda\left(mm^{-1}\right) = \gamma + \beta\mu + \alpha\mu^2. \quad (14)$$

Combining the above Λ-μ relationship with Eq. 9 we derive the following equation for μ:

$$\gamma_1 + \beta_1\mu + \alpha_1\mu^2 = 0. \quad (15)$$

where, $\alpha_1 = \alpha D_0$, $\beta_1 = \beta D_0 - 1$ and $\gamma_1 = \gamma D_0 - 3.67$.

Solving the above quadratic equation we get two solutions, from which we select the one that μ is within the physically acceptable range of -2 and 12. Figures 1a, 1b and 1c show D_0-Z_{DR}, N_W-(Z_H, Z_{DR}) and Λ-μ scatter plots calculated from raindrop spectra overlaid by the corresponding best-fit relationships.

3.2 The β-method

The method starts with estimating the β parameter of the linear axis-ration model from the non-attenuated radar parameters (Z_H, Z_{DR}, and K_{DP}) using the algorithm described by Bringi et al. (2002), and modified by Park et al. (2005) for *X*-band frequency using DSD spectra from Japan:

$$\beta = 0.9425\left(\frac{K_{DP}}{Z_H}\right)^{0.2624}(\zeta_{DR} - 1)^{0.377}. \quad (16)$$

where $\zeta_{DR} = 10^{0.1Z_{DR}}$ is the differential reflectivity in linear units. The method uses the estimated β parameter to retrieve D_0 and N_W from polarimetric radar parameters exceeding the thresholds of 0.2 (°km^{-1}) for K_{DP}, and 35 dBZ and 0.2 dB for Z_H and Z_{DR}, respectively. Those relationships are shown below.

$$D_0 = 0.627 Z_H^{0.057} \zeta_{DR}^{\left(0.03\beta^{-1.22}\right)}. \quad (17)$$

$$\log_{10} N_w = 2.97 Z_H^{0.07} \zeta_{DR}^{\left(-0.03\beta^{-1.26}\right)}. \quad (18)$$

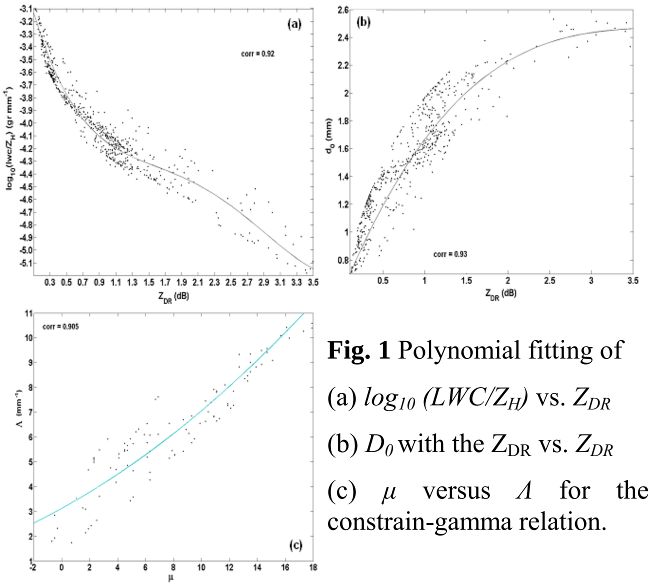


Fig. 1 Polynomial fitting of

(a) log_{10} *(LWC/Z_H)* vs. Z_{DR}

(b) D_0 with the Z_{DR} vs. Z_{DR}

(c) μ versus Λ for the constrain-gamma relation.

For $Z_H < 35$ dBZ and $Z_{DR} \geq 0.2$ dB the relationships for N_W and D_0 are functions of (Z_H, D_0) and (Z_{DR}), respectively, while for $Z_H < 35$ dBZ and $Z_{DR} < 0.2$ dB the corresponding relationships are functions of (Z_H, Z_{DR}) and (Z_H, N_W). For estimating the μ parameter we use the constrained μ–Λ relationship as was described in the previous method, which is a modification to the original approach described in Bringi et al. (2002).

4 Algorithm evaluation

4.1 XPOL vs. S-POL comparison

During the International H_2O Project (May 16 up to June 22nd of 2002) we operated the National Observatory of Athens *X*-band polarimetric radar (XPOL) in coordination with NCAR's S-POL to measure mesoscale convective systems. We use here coincident XPOL/S-POL observations to evaluate the XPOL DSD retrievals using as 'reference' corresponding DSD parameters derived from S-POL measurements. The well-established *S*-band polarimetric technique of Brandes et al (2004) is used for this purpose. In Figure 2 we show a sample ray taken from the June 16 case study in IHOP. The figure consists of two three-panel plots illustrating the DSD parameter retrievals (D_0, $log_{10}N_W$ and μ) from XPOL using the two different approaches we discussed in section 3. The light-gray and dark-gray dashed lines are retrievals from the attenuation-corrected XPOL radar observations. The dark-gray corresponds to the *constrain*-method estimates while the light-gray line to the β-method. The following points are noted. First, the β-method exhibits greater variability in N_W and D_0 estimation than the other technique. This variability is significantly outside the range of the S-POL retrieval. For example, the β-method significantly overestimates D_0 at the storm peak of the ray, while the *constrain*-method seems to match better the S-POL retrievals with higher correlations (0.85, 0.83 and 0.9 for D_0, N_W and μ, respectively).

We notice large biases for the β-method. Specifically, for in the ranges of 18 to 26 (km) we notice an N_W overestimation of the order of 1 ($mm^{-1}m^{-3}$) and at two consequently ranges (22 and 25 km) an underestimation of 0.5-1 ($mm^{-1}m^{-3}$). Similar deviations we observe in D_0 and μ. On the other hand, the *constrained*-method is following much closer the reference S-POL data. In summary, the bias and root-mean-square difference statistics of the two techniques against S-POL parameters are 0.22 and 0.26, respectively, for the *constrain*-method and for the β-method, 0.23 and 0.3, respectively.

4.2 XPOL vs. disdrometer comparison

In this section, we evaluate the two *X*-band DSD retrieval algorithms on the basis of coincident *X*-band polarimetric radar and *in situ* disdrometer data. The data originate from measurements of a Typhoon pass (August 9, 2003) over Japan made jointly by a dual-polarization *X*-band radar (MP-X) and a near range (18 Km) JW disdrometer.

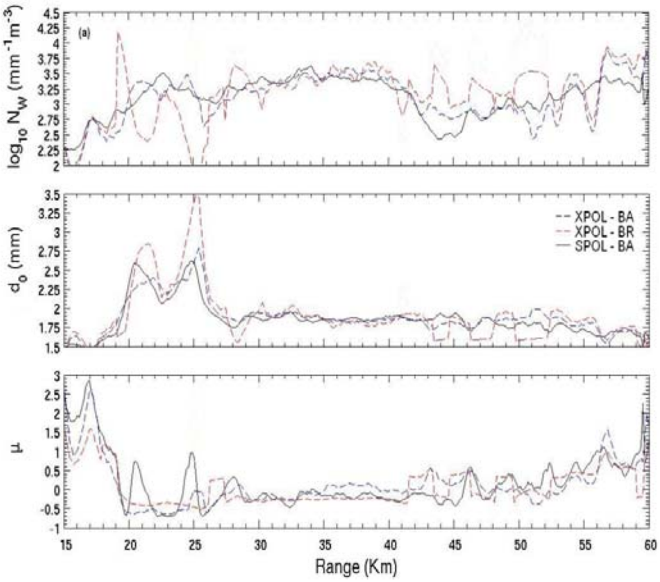


Fig. 2 A sample ray plot illustrating a good comparison of the two DSD methods with coincident X-/S-band measurements.

For the evaluation of the two methods we use here time series plots (see Figure 3) of *X-band polarimetric* DSD retrievals based on the two techniques and DSD parameters calculated from the *in situ* disdrometer data. In Figure 4 we show frequency histograms of those parameter estimates. Both figures show a good agreement between the two XPOL radar retrieval methods and the parameters derived from DSD spectra. Again here the *constrain*-algorithm exhibits closer agreement to the DSD spectra-derived parameters relative to the β-method. In summary, the correlation, bias and root-mean-square difference statistics for D_0, *Nw*, and μ parameters for the two methods against DSD spectra-derived parameters are: [correlations: 0.88, 0.78 and 0.57; biases: 1.03, 0.99 and 1.65, standard deviations: 0.45, 0.47 and 5.45, respectively], for the *constrain*-method and for the β-method are: [correlations: 0.67, 0.56 and 0.46, biases: 0.94, 0.98 and 0.64, standard deviations: 0.56, 0.59 and 6.01, respectively].

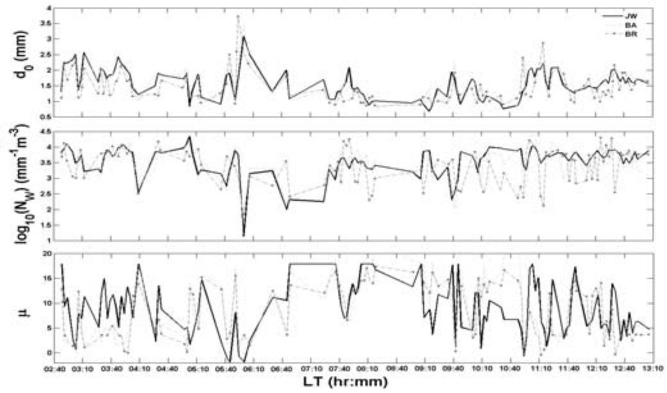
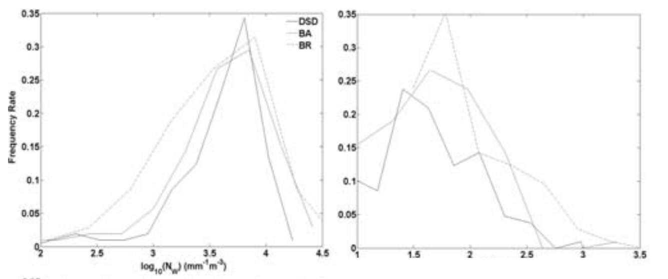


Fig. 3 Timeseries comparison of the three DSD governing parameters of the Gamma distribution predicted from radar observations using the two DSD retrieval methods (*constrain*-method light gray dashed line; and β-method dark gray dashed line) and calculated from measured DSD spectra.

The frequency histograms confirm the better agreement of the *constrain*-algorithm (in this plot indicated with BA) with the disdrometer spectra-derived parameters versus the β-method (in this plot indicated with BR). The mode of the D_0 and N_W are at about 1.8 and 3.8, respectively. The μ parameter also exhibits a good agreement between the *constrain*-algorithm and disdrometer data. The β-method gives high concentration of μ parameter at two ranges (2-4 and 13-15), which is not in agreement with the other two datasets.



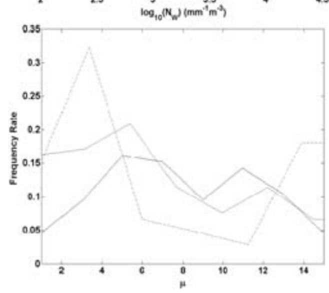


Fig. 4 Frequency histograms of $log_{10}N_W$ ($mm^{-1}m^{-3}$), D_0 (mm) and μ parameters for horizontal polarization reflectivity for values greater than 20 dBZ.

5 Summary and conclusion

The long-lasting goal of dual-polarimetric radar has been the rain rate estimation and the estimation of the parameters of the corresponding raindrop size distribution. *X*-band radars even though are associated with significant rainpath attenuation are more sensitive in the low-to-moderate rain rates in favor of *S*-band radars.

In this paper we presented and evaluated two algorithms for the estimation of the three-parameter gamma *DSD* model, D_0, Nw, and μ from *X*-band polarimetric radar observations, Z_H, Z_{DR}, and K_{DP}.

Overall, our analysis showed that the β–method is unstable in low-to-moderate rainfall compared to the *constrain*-algorithm because it strongly depends on K_{DP}. On the other hand the *constrain*-algorithm avoids the use of simulated DSDs and the errors associated with K_{DP}. Research is in progress to expand quantitative comparisons of *X*-band DSD retrievals with DSD measured spectra from different radar ranges and precipitation categories.

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