AQUARadar

Rain estimation from partially filled scattering volumes

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Introduction

Atmospheric parameters retrieved by remote sensing techniques often represent averages over considerable space or time intervals. Unresolved variability of corresponding primary physical values is a problem as soon as non-linear relations between the primary physical value and the retrieved parameter come into play, as for example the *R*(*Z*)-relation between radar reflectivity *Z* and rain rate *R*. While this is recognized to be a crucial issue for many space based techniques (particularly for cloud and rainfall retrievals) due to the large sensor footprints (Durden et al. 1998, Heymsfield et al. 2000), ground based radar measurements of rainfall are sometimes considered as a reference having sufficient resolution with negligible unresolved variability. On the other hand precipitation is known to show strong variability even on very small scales. There is in fact evidence that no scale exists for spatial hydrometeor distributions, where Poisson statistics would be an adequate description analogue to molecules of a gas in thermodynamic equilibrium (Jameson, 2005). Radar calibration using distrometer-data is therefore faced with the problem of non-matched spatial resolution of distrometer- and radar-data respectively.

Definition of inhomogeneity of rain drop number density

Due to the stochastic position of hydrometeors in space, the term "inhomogeneity" of drop density and accordingly of the radar reflectivity must be defined in a statistical sense.

If hydrometeors would be distributed in space like molecules of a gas in thermodynamic equilibrium (Poisson-distribution), the probability density function (pdf) of the single-shot radar-reflectivity would follow an exponential probability density function (e.g. Doviak and Zrnic, 1993). The exponential distribution has the well known property, that its dispersion σ is equal to its mean value $\overline{Z}$. Therefore we introduce the ratio $\overline{q} = \sigma / \overline{Z}$ as "inhomogeneity" parameter. The value $\overline{q} = 1$ corresponds to thermodynamic equilibrium, which is here synonymous for "homogeneous" distribution. Any deviation from homogeneity represents clustering or "inhomogeneity" and leads to $\overline{q} > 1$.

Exponential pdf of radar reflectivity from hydrometeors, which are Poisson-distributed in space:

$$p(Z) = \frac{1}{\sigma}\exp(-Z/\sigma)$$

Estimates of $\overline{Z}$, σ and q, indicated by Z_e, σ_e and q_e respectively, can be obtained by sampling Z in n different volumes calculating

$$Z_e = \frac{1}{n}\sum_{i=1}^{n} Z_i \qquad \sigma_e = \left(\frac{1}{n-1}\sum_{i=1}^{n}(Z_i - Z_e)^2\right)^{0.5} \qquad q_e \equiv \frac{\sigma_e}{Z_e}$$

Observation of inhomogeneity

Radar data from a ring between 4 and 5 km radius were used, in order to minimize artifacts due to potential range dependencies of the retrieved radar reflectivity. The pixels represent 60 m range-, 2° azimuth-sector-, and 30 s time-intervals. Not only the mean values but also the standard deviations of the single-shot/single- range-gate reflectivities were recorded. Each pixel consists of about 1400 samples, but the number of *independent* samples n_f is considerable smaller, due to the antenna beamwidth of 2.5° and the finite coherence time of the signal. A lower limit is $n_f = 48$, which is equal to the product of range gates and antenna revolutions included in the small pixel.
In order to simulate the resolution of typical weather radar data, groups of 96 pixels (henceforth called *small pixels*) were integrated to *large pixels* corresponding approximately to squares of about 1 km² area (figure 1).

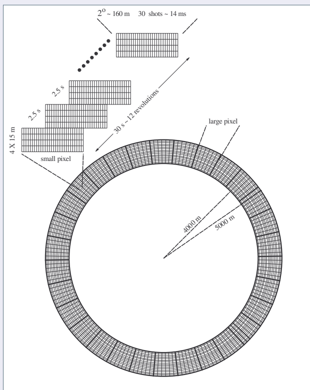

Figure 1

The small-pixel data were used to calculate the "small-pixel inhomogeneity parameter" $q_s \equiv \sigma_s/Z_s$. q_s is plotted versus the radar reflectivity in figure 2. The detection threshold is at about 0 dBZ. One recognizes that q_s is centered around 1 for radar reflectivities below the detection threshold as to be expected for stationary noise.

Above the detection threshold q_s increases, assuming a mean level at about $q_s = 2$.

The distributions of q_s look similar in figures 2a and b, which correspond to prevailing stratiform and convective rain events respectively.

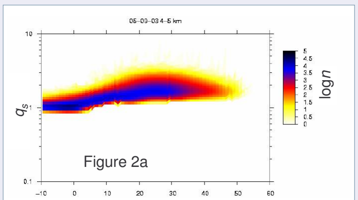

Figure 2a

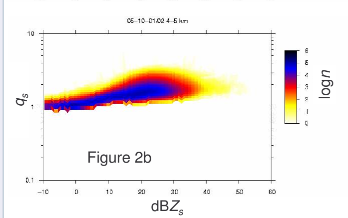

Figure 2b

The effect of sub-resolution inhomogeneity

The sub-resolution-inhomogenity of large pixels, q_l was calculated and the effect on the rain rate retrieval was investigated by alternative rain rate calculation $R(Z_l)$ and $R_l(Z_s)$, assuming a standard $Z = aR^b$-relation with $a = 250$ and $b = 1.6$. The ratio q_r of both rain rates is plotted in figure 3a and 3b for the same rain events as in figure 2a and 2b. As σ_l is based on the variance of mean values Z_s, here the homogeneous case corresponds to $q_l = 0$.
For definitions, see equations below.

$$Z_l \equiv \frac{1}{l}\sum_{i=1}^{l} Z_{s,i}, \quad \sigma_l \equiv \left(\frac{1}{l-1}\sum_{i=1}^{l}(Z_{s,i} - Z_l)^2\right)^{0.5}, \quad q_l \equiv \frac{\sigma_l}{Z_l}$$

$$R(Z_l) \equiv aZ_l^b, \quad R_l(Z_s) \equiv \frac{1}{l}\sum_{i=1}^{l} R(Z_{s,i}), \quad q_r = \frac{R(Z_l)}{R_l(Z_s)}$$

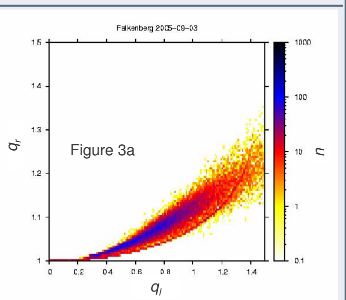

Figure 3a

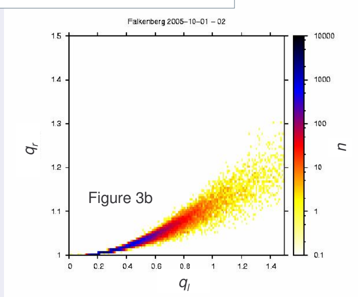

Figure 3b

Conclusion

It could be shown that the relation between radar reflectivity and rain rate depends on the resolution, if the spatial drop distribution is inhomogeneoeus ($q_l > 0$). Spatial integration of Z leads to higher rain rates than spatial integration of R, when using a standard Z-R-relation. The quantitative effect can be parameterized with q_l. For operational application it would be necessary to establish a parameterization, which does not rely on sub-resolution observations.

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